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This research effort in the field of software reliability is primarily based on probability: as applied to error detection rates; as applied to the generation of input test data; and as applied to the economics of random versus constructed test cases. The background description of the flow of computations is made by means of a directed graph representation and a connection matrix depicting possible links between program segments. Random numbers selected from a given domain by several distributions are employed as input to programs instrumented to detect the use of the program's segments. An

20. Abstract (Continued)

algorithm is employed to estimate the total number of execution sequences which are likely to be eventually driven by random number inputs, and, by implication, the point in testing where the change to constructed cases should be made. Models for determining the count of these execution sequences are described and tables which facilitate estimation of the number of logical paths and related parameters are provided for convenience.

PREFACE

This report documents the results obtained under contract F44620-74-C-0008 entitled "Quantitative Methods for Software Reliability Measurements", during the 36 month period ending in November 1976. This work was conducted in the Information Systems Sciences Branch, of the Data Control and Processing Subsystems Department, McDonnell Douglas Astronautics Company-West, in Huntington Beach, California. Excepting the Appendix I, this report was written by Paul B. Moranda, Information Systems Advisor Senior and Principal Investigator, under the direction of Zygmunt Jelinski, Program Manager. The assistance of Mrs. Carolyn Boettcher of McDonnell Douglas Automation Company, in performing several special numerical analyses and in writing the supporting computer programs for the study, is gratefully acknowledged. This work was conducted under the auspices of the Air Force Office of Scientific Research and was monitored by Lt. Col. Thomas Wachowski and Lt. Col. George W. McKemie whose assistance, both direct and indirect, are also gratefully acknowledged.

ABSTRACT

This research effort in the field of software reliability is primarily based on probability: as applied to error detection rates; as applied to the generation of input test data; and as applied to the economics of random versus constructed test cases. The background description of the flow of computations is made by means of a directed graph representation and a connection matrix depicting possible links between program segments. Random numbers selected from a given domain by several distributions are employed as input to programs instrumented to detect the use of the program's segments. An algorithm is employed to estimate the total number of execution sequences which are likely to be eventually driven by random number inputs, and, by implication, the point in testing where the change to constructed cases should be made. Models for determining the count of these execution sequences are described and tables which facilitate estimation of the number of logical paths and related parameters are provided for convenience.

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I. INTRODUCTION AND OVERVIEW

A. Introduction

A reasonable categorization of the broad study of software reliability corresponds in many respects, to one of the commonly employed categorizations of the phases of development which software packages undergo. Corresponding to the design phase of software development, there is an aspect of the reliability problem dealing with estimation of the error content before (a priori) any actual running of the program has taken place. The test phase of software development can be associated with reliability estimates which are determined during relatively exhaustive testing by measures of the internal operations of single modules; some measure of the complexity of the package and of the response to randomly chosen inputs are useful in developing reliability estimates. The third phase of software development can go under the name of user-phase; this phase would include any significant testing of sets of modules (ordinarily this makes up part of what is called integration); this phase would employ the time record of the occurrences of errors, for the primary data for system level reliability estimation.

Data relating to the third category was the object of study by MDAC during the period prior to the contract interval (1971-1973) and as a result of this effort, the first parametric model for the software debugging process was developed and reported in the literature (Reference 1). This model required no knowledge of the actual coding, the flow charting, or the mode of operation, and, indeed, treated the software package as a "blackbox".

The first category, that of developing a priori estimates, vis a vis the a posteriori estimates of the previous work, was the principal subject of the work performed during the first year of the contract. In order to develop estimates of that type it was necessary to "look inside the box," and to develop efficient means of describing the information. The resultant

study resulted in a directed graph/connection matrix representation for the static and dynamic analyses of the test programs.

Input test cases, formed by means of random number generators, were employed to drive the programs in different ways.

As an outgrowth of this research, particularly that which developed the framework for computation of a priori or "operational" software reliability estimates, a need for deeper investigations into test case selection was noted and redirection in that area was carried out during the second phase.

As an aid to determining the point in testing where a change should be made from tests based on random number inputs, to tests constructed on the basis of the code itself, an algorithm was developed which permits evaluation of the "yield" which can be gained by additional testing.

B. Objectives and Task Descriptions

The original plan for the research redirected in ways subsequently described, was given in terms of three tasks of the first (of three) phase(s):

> Task I - to investigate the existence of relations between the number and range of input variables and software reliability, and to establish a relationship between the gross measures (such as program size, number of branching statements) of a software package and its reliability. A program testing translator will be used as a tool to analyze frequency of execution of branches and instructions.
>
> Task II - to develop an algorithm representing relationships between variables and the software reliability - extrapolation techniques will be developed which will relate the counts obtained over a given collection of subsets or samples to the counts which would be obtained over larger sets.
>
> Task III - to generalize the above algorithm. This algorithm will be tested on a representative sample of FORTRAN programs and adjusted accordingly.

C. Course of the Research Program

First, it is well to note that, with respect to the problem of relating reliability to measures of the internal parameters of a program, that two separate studies by other groups, (one by TRW, Reference [2] and one by Lulejian and Associates, Reference [3],concluded that out of 22 quantitative measures of programs (and their programmers) there was only one case which showed a significant correlation. This significant correlation was between reliability and program size. Only the contrary would be surprising - more opportunities for errors should be accompanied by more errors. These studies employed relatively large real time programs (command and control) and as such they dealt with programs which are quite relevant to this study. Because of the essentially negative results which were obtained, there seemed to be little use in pursuing similar lines in our investigation.

The initial work focused on the instrumentation (insertion of monitoring instructions) of programs so that their dynamic (performance) rather than their static (structured) aspects could be examined. The investigation of the dynamic or operational characteristics turned out to be important: in the final analysis it is relatively unimportant how complex a program appears to be, it matters more what the program actually does. The instrumentation was accomplished by use of a MDAC-developed software tool called the Program Testing Translator (PTT). This tool described in some detail in Section of this report (and in complete detail by L. Stucki in Reference (4)), provides the user with a means of establishing the usage of each instruction and each branch of predicates of FORTRAN programs. In addition, the range of values which the program variables take as a result of a particular value of the input variable (which can be considered to be a vector, since in most programs several input variables are employed) was recorded for each segment containing an assignment instruction, the equivalent of a program function.

Any particular point of the input data set, more commonly called the domain of the input variable, will cause the associated program to sequence its instructions in a particular way. When the program has been instrumented with the Program Testing Translator, data relating to the path of computation and values achieved by computed variables can be composed. An augmentation

of the PTT permits automatic generation of program segments and this materially aided the analysis.

After the initial experiments were made with random numbers with two programs, there were several factors which led the investigation to open up new avenues of research. In the first place, a review of the International Mathematical Subroutine Library (IMSL), which had been selected as the set of programs for the investigation, revealed that most of the inputs to these programs cannot realistically be assumed to be governed by probability laws; the degree of a polynomial and the order of a matrix are not random for example, and even when the variables can be considered random, such as the coefficients in a polynomial root locator, the law which fits the sampling of these numbers (by the universe of all users) will not be known. The alternative of analyzing real-time programs requires special knowledge of special languages and special machines and so requires a number of highly skilled programmers.

In addition, the measurement of reliability of a program in the sense used in the task description, depends on data which, it was assumed would be recorded either by the users of the routines of the library (it was assumed they would record anomalies as they were discovered), or, alternatively, by a single user or small set of users, who could pervasively test the programs by means of random number generators and special tests until a significant set of errors were discovered. Neither of these turn out to be practical alternatives.

On the one hand, the subroutines from the software library which are used, are generally in linkage with others, and the "input" data to the routine usually is totally transparent to the programmer, it comes as a result of prior computations. Of course, it is possible to gain snapshots of this inputs, but the price is considerable coding time by systems-level programmers; it is unlikely that this overhead can be justified.

On the other side, it becomes apparent,with experience, that the input per se is less a determinant of the degree of testing than such things as the structure of the program or the amount of protection inserted to guard against certain, perhaps rarely occurring, events. Furthermore, it is likely that a user who did not participate in the design of the program

will be essentially naive with respect to the occurrence of an actual error and with respect to the location of the "sensitive" portions to test with well-chosen input data.

Because of the factors cited above, a fresh approach was taken; one which, to begin with, was essentially exploratory. The major aim in this approach was to develop an a priori measure of reliability using input data which is assumed to be randomly chosen by a known law or set of laws, and the length and connectivity of the segments (the programs building blocks) which the random data drive. An important feature of this type of testing is that the dynamic (performance) rather than the static (structural) aspects are emphasized. It is important also to note that the output of the program is not examined critically and consequently most software errors which do occur will not ordinarily be detected. The a priori reliability is developed on the basis of the way the program is used and a "universal" a priori probability for the rate of occurrence of coding errors.

This type of measure would be valuable in some real time programs which depend on sensor measurements for some or all of its data. Gyroscopes, telescopes, accelerometers, and their pickoffs, resolvers, and similar devices all have random components in their associated measurements; furthermore, the systematic portion of the measurement in many cases may be considered in a larger sense as having a probability law governing their choice - initial vector heading at alignment time might, for example, be any value (uniformly) on the circle.

As this study developed, it became clear that the use of random numbers as input can serve very well toward achieving comprehensive testing of almost any program. This investigation then picked up earlier work, started during the exploratory phase, and simple measures of the degree to which random numbers can test a program were developed. Following this, there was an attempt made to give a quantitative measure to the eventual level of testing which could be achieved by random number inputs. This resulted in an algorithm which was even more useful than the one which was sought: it provided an estimate of the total number of execution sequences (realizable logical paths) which can be achieved by random testing. The application of this algorithm on two programs produced results which were surprising

in the respect that the numbers produced were orders of magnitude smaller than those propounded by the "conventional myth". Further, the results were consistent in the sense that application of the algorithm to a portion, an initial segment, of data produced estimates which the later realization tended to support.

D. Publications and Presentations

The following papers which were sponsored entirely, or in part, by the contract are listed below. In most cases these were personally presented to a professional audience and published in proceddings of the respective meetings.

1. "Predictions of Software Reliability During Debugging", 1975 Proc. Annual Reliability and Maintainability Symposium, Washington, D.C., January 1975.

2. "Estimation of A-Priori Software Reliability", Proceedings of Computer Science and Statistics: 8th Annual Symposium on the Interface, Los Angeles, February 1975.

3. "Software Reliability Predictions", International Federation of Automatic Control 6th World Triennial Congress, Boston/Cambridge, August, 1975.

4. "Probability-Based Models for the Failures During Burn-In", Joint National Meeting ORSA/TIMS, Las Vegas, November 1975.

5. "A Comparison of Software Error Rate Models", Fourth Texas Conference on Computing Systems, Austin, Texas, November 1975.

6. "A Failure Rate Model for Burn-In through Steady State", Joint National Meeting of ORSA/TIMS, Philadelphia, March 1976.

In addition, the manuscript for a book to be published by Academic Press has been prepared and is in the preliminary stages. This book, "Probability-Based Models for Software Reliability Analysis", authored by P. Moranda and Z. Jelinski, contains much material developed under the AFOSR contract and described herein. It is not current in the respect that several new and significant results produced under the contract are not included.

II. PRELIMINARY TECHNICAL DISCUSSION

A. Framework of Representation

In the customary renditions of program flowcharts, each (rectangular) block represents either a simple instruction, or a group of operations, with a single output, while each diamond represents a single explicit or implied predicate which has two or more output options. Connecting the blocks and diamonds of a flowchart, are directed lines denoted, and referred to, as arrows. These lines represent the options possible and are called flow-of-control arrows. These fundamental building blocks are adequate for the static or structural description of a program, but are not convenient for representing its operational aspects. The basic operations are better defined in terms of some simple program components. These lend themselves to mathematical descriptions and they motivate the choice for the "atomic" or fundamental unit of description.

First, it is noted that an instruction in a program, while easy to define (statically) in "machine language", becomes rather difficult in most of the higher order languages. Thus a "clear and add" instruction, in machine language, causes a register (accumulator) to be set to zero and another register to be transferred to the cleared register and nothing more. Once the final bit is transferred, the machine waits until the next instruction, which is generally started by a timing or clock pulse. On the other hand, the concept of an instruction in the higher languages is less clear. An "instruction" in ALGOL, for example, is either a <u>statement</u> or a <u>declaration</u>, and in either case is used to indicate required compiler (as against computer) actions. As a result of compiler action, an object program with actual instructions, is produced, and it is in proper form for computer execution.

Thus, there is a spectrum of statements in that language: the simplest type is an assignment, such as X:=1; while one of the more complex statements is, *begin ... end*, which group statements together to form compound statements (and blocks).

In any higher order language where grouping is required, there is a need for so-called delimiters (explicit or implicit) which can be used as boundaries for the steps, and form the building blocks of a program. A similar device is required in the description of dynamic operations - a means of grouping instructions into fundamental operational units.

Generally, the linking of instructions can be represented by means of a Boolean indication, with the value 1 used where the instructions are or can be "contiguous", and 0 used to denote the fact that they are not connected. These Boolean values could be used as entries of a connection matrix whose row and columns are numbered to accord with an (arbitrary) numbering scheme for the steps. But a straightforward application in this manner, on the instruction level, would normally produce inordinately large and unmanageable connection matrices. Some of the redundant information in such a matrix could be eliminated if certain agreements can be made: for example, if step 1 is always followed in sequence by steps 2, 3, and 4 and there is not opportunity for branching until step 4 (at least), then steps 1 through 4 can be merged or combined, and three of the rows and columns of the connector matrix could be eliminated. This reduction in redundancy is an additional reason for choosing groups of instructions for the description.

Because certain instructions or statements have more than one output(such as *if...then...else*) there is a need to devise a convention which will permit identification of each of the exits. If statement A is a single-output statement and it connects to statement B which has multiple outputs, the notation [A,B), which is "closed" on the left and "open" on the right, is meant to imply that A is executed and control is passed to (or toward) B, but that B is not executed, but it is next in line. If B is a two-output instruction and connects to L_1 and L_2, then both $[B,L_1)$ and $[B,L_2)$ are used to describe the optional branches which can be taken.

The procedure which has been described can be so far, by a flow diagram of a very simple program. In Figure 1 is a combination of a code listing on the right and a flow diagram on the left. Numbers refer to the instructions listed. The program is designed to process a sequence (one or more) of

lists, with each list consisting of "test scores" augmented by the number -1 (which is not a test score); the last list is further augmented with a -2 (for HALT purposes). The program tallies the number of scores within each list which are at least as large as 70 (passing), and also tallies the total number of passing scores within all lists (the Grand Sum).

To continue with the description, it will be seen in Figure 1 that the first connection to a branching instruction is made at instruction number 3. From 3 the branch taken is determined by the predicate (X=-2) and how the input to 3 (carried out of 2) values it (true or false). Thus, instruction number 3 is connected to 14 and to 4, as potential (operating) successors to 3. In the same way, 5 as a branching statement connects to 6 and 10.

A variation of the technique which is usually employed, characterized by connecting "nodes" (representing sets of instructions) is proposed here. Emphasis in this variation is on the branches which emanate or terminate with branching instructions, and, in fact, the fundamental or "atomic" element in the representation of a program is taken to be a <u>segment</u> or string of instructions between two branching instructions. More precisely a segment is: a sequence of instructions starting with either a START, or a branching instruction, and ending (but not inclusively) with the first subsequent branching instruction, or a HALT, in which particular case the segment is considered to include the instruction which ends it.

As an example of the way segments are developed, the flow diagram in Figure 1 is analyzed:

S_1 = [1,2,3)
S_2 = [3,14,15]
S_3 = [3,4,5)
S_4 = [5,10,11,12,13,3)
S_5 = [5,6)
S_6 = [6,8,9,5)
S_7 = [6,7,8,9,5)

The distinction between brackets and parentheses is important and has been noted. The only cases where square brackets are used on the right are those in which the last instruction listed is a HALT (number 15 in the example).

Any particular set of values (for the coordinates) of the input vector (point in the input space), causes exactly one sequence of operations to be executed. These segments linked together form a logical path through the program.

It is useful to modify the term logical path with the word realizable when input data can cause it. Before data is entered, possible (or feasible) logical paths can be formed by any concatenation of contiguous segments which have the START-segment first and end with a HALT-segment. In the case a program has self-contiguous segments (loops) or one or more concatenations which join end-to-end, the number of (possible) repetitions of the joined segments is arbitrarily large - except where a predetermined number of traversals are specified in the program.

The following sequences of segments in the program of Figure 1 are illustrative of some possible or feasible logical paths:

S_1S_2
$S_1S_3S_4S_2$
$S_1S_3S_4S_3S_4S_3S_5S_6S_4S_2$
$S_1S_3S_5S_6S_4S_2$
$S_1S_3S_5S_7S_4S_2$

The first path is of minimum possible length, linking, as it does, the START - and HALT - segments. The last two are interesting in that they exhaust the collection of segments.

In order to determine realizable logical paths, the documentation or "program writeup" must considered. In this simple case it is very easy to establish data which will realize the flows represented by the last two sequences of the above list. (It should be noted that insofar as testing to the instruction-level only one of these two need be driven but to obtain segment or branch-level testing, both need to be tested).

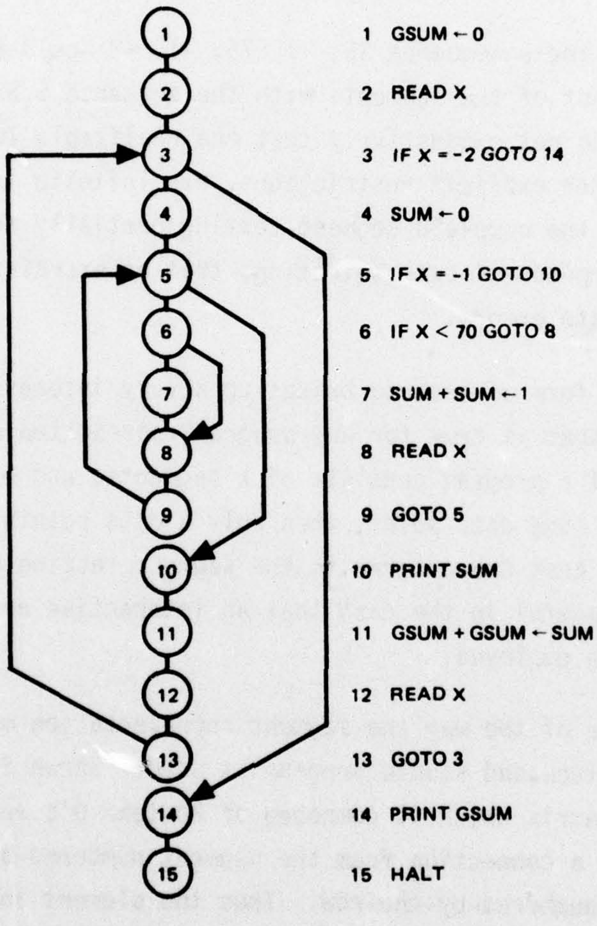

Figure 1. Test Scores Program and Flow Diagram

If for example the data sequence (stacked)

$x = 35, -1, -2$

is employed, the next to the last sequence of the above list describes the flow, and for the "stack"

$x = 75, -1, -2$

the last sequence describes the flow. The two stacks together provide an exhaustive test of the segments of the program.

Moreover, a single sequence 35, -1, 75, -1, -2 would also produce an exhaustive test of the segments with the sequence $S_1S_3S_5S_6S_5S_7S_4S_2$. While these do not exhaustively test the realizable logical paths (which, without further explicit restrictions, are infinite in number), it is well to note that the complete segment-testing partially accomplishes one of the major purposes of case selection, that of exercising all instructions so as to locate errors.

This limited form of testing brings up a very interesting and very obvious observation that is true for any program represented as a collection of segments: if a program consists of k segments, and every segment can be exercised by some data point, then only k data points are required to exhaustively test the program in the segment testing sense. This is of course very useful in the case that an interactive or communicative mode of testing is employed.

As an example of the way the segment representation might be used, the previously discussed simple program is used. Shown in Figure 2 is a connection matrix which is composed of Boolean 0's and 1's, with a 1 representing a connection from the segment numbered-by-the-column to the segment numbered-by-the-row. Thus the element in the 4th column and 2nd row has a Boolean 1 since the segment S_4, connects (or more properly can connect) with segment S_2, as shown in Figure 2.

The Boolean Matrix Algebra is clear and the essential rules are shown at the bottom of Figure 2. These rules are formed directly from the basic Boolean Arithmetic. As an illustration of the use of the algebra for constructive testing, the concept of "basis" or state vectors is employed.

FROM

TO	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	1	0	0	1	0	0	0
3	1	0	0	1	0	0	0
4	0	0	1	0	0	1	1
5	0	0	1	0	0	1	1
6	0	0	0	0	1	0	0
7	0	0	0	0	1	0	0

CONNECTION MATRIX

ALGEBRA

$1 \oplus 1 = 1$ $\quad$ $0 \oplus 1 = 1$

$0 \oplus 0 = 0$ $\quad$ $1 \oplus 0 = 1$

$1 \otimes 1 = 1$ $\quad$ $1 \otimes 0 = 0$

STATE VECTORS

$$E_1 = (1,0,0,0,0,0,0)^T$$

$$E_2 = (0,1,0,0,0,0,0)^T$$

Figure 2. Connection Matrix and Algebra

There are as many state vectors as there are segments and they are denoted by E_1, $E_2, \ldots, E_m$ where E_i is the transpose of a row array consisting of 0's, except for the ith position, which contains a 1. These vectors can be used as markers or tokens to represent the location of the computing operation at the "initial" computing time; E_3 for example, could be used to show that at some arbitrarily chosen time, the site for computing is in segment 3. When E_3 is multiplied by the connector matrix, C, the arithmetic shows that the vector $[0,0,0,1,1,0,0]^T$ results. This vector can be represented as the vector sum E_4+E_5 and would represent the fact that in the first exit from E_3, the computing can be carried forward in either segment S_4 or segment S_5.

(In an interactive mode this might be used as follows: if the data point which is used to "feed" the program has previously caused S_3 and S_4 to be exercised, the adaptive user, or tester, would search for a new area of the input data space, in an attempt to exercise segment S_5 in some subsequent step in the testing process).

The connection matrix idea has been proposed in other applications, notably by F. E. Hohn and L. Shissler [5] in which it is applied to hardware switching circuits. The theory is well developed by H. G. Flegg in his book [6].

The algebra of Boolean matrices and vectors is exhaustively treated by Flegg, and, although there is a slight difference between the matrices which depict software and those that depict hardware (each hardware circuit node is generally considered to be connected to itself, while in software this is true only for certain kinds of loops), the algebras are not significantly different. The rules of multiplication and addition are the same, the basic associative and distributive laws hold and all of the ordinary matrix algebra holds.

Although the basic rules and many others which are introduced with the concepts of order, complementation, zero, and identity are easy to verify, the requirements for any of these more extensive rules are essentially non-existent. (Some 50 rules have been verified but have little promise in our applications). Of interest to software applications are some of the simple-to-describe techniques involving multiplication. One of these has been noted above where the state

vectors are "propagated." In a more interesting application, it is easy by matrix manipulation to find if there is an eventual link between a given segment and another in a program which has n segments. It is only necessary to "raise" the nxn matrix to the first n powers. The remote, or subtle, i.e., multi-step connection, if it exists, will be manifested by a Boolean 1 in the position corresponding to the 1st order connection between the two segments. As a matter of fact and interest if ordinary (real number) matrix multiplication is used instead of Boolean algebra, the number which occurs after, say, k multiplications represents the number of different paths that join the two segments in exactly k steps.

C. V. Ramamoorthy has developed the basic technique (using real numbers as well as Boolean symbols) to some very interesting results. In the majority of applications Ramamoorthy [7] employs the concept of a generating function from node i to node k. For a given connection matrix, C, the generating function is

$$G_{ik}(z) = \sum_{m=0}^{\infty} g_m z^m$$

where explicitly

$$G_{ik}(z) = (I-Cz)^{-1}$$

where I is the identity matrix and z is a dummy variable. Under the non-Boolean interpretation (i.e., real numbers) of the connection matrix, C, as an nxn matrix, the factor g_m represents the total number of ways of reaching node k from i in m-steps.

Although it should be stated at this point that the apparent promise of achieving useful results in studies of software has not been met, for reasons which are described later, it is well to note what can be done by developing the fundamental concept. Several important theorems, lemmas and observations, derive from the use of the generating function and its companion function, called the characteristic function $|I-Cz|$, where the vertical lines denote the determinant of the matrix I-Cz. (It is noted that this concept is used in ordinary matrix theory; the characteristic function determines the so-called eigenvalues of the matrix).

It is worth noting that the sophistication offered by the generating function does not seem to produce any result not otherwise obtained with as much facility by the connection matrix and Boolean operations as defined above. Nonetheless, a review of Ramamoorthy's findings are described in terms of the characteristic and generating functions.

One of the easily obtained results is that $G_{SE}(z)$, (S for start and E for end) is equal to zero if and only if there is no path linking S to E. If the coefficients g_m are accepted as the number of ways of reaching E from S this is clear: otherwise, the aforementioned technique employing powers of the connection matrix can be used.

Because a directed graph which has no single-step loops has zeros on the diagonal of the connector matrix, and, further, must have at least one of the matrix elements $C_{\mu\nu}$ or $C_{\nu\mu}$ (the symmetric-about-the diagonal elements) equal to zero, the determinant $|I-Cz|$ will be constant, so that the inverse matrix, which is formed by taking the quotient of cofactors of the matrix and this determinant, consists of polynomials of degree at most n. Thus,

$$G_{SE}(z) = \sum_{i=0}^{n} g_i z^i$$

for loop-less graphs. For the general case the characteristics function is a polynomial and $G_{SE}(z)$ is an infinite "series". So the above shows that the largest exponent of the generating function is finite when there are no loops. The converse is also true since g_m will be non-zero if there are loops.

The concepts of a strongly connected, and maximally strongly connected graphs and subgraphs are due to Ramamoorthy [8]. A graph (or subgraph - one obtained from a given graph by taking a subset of its nodes and retaining all of the connections between these nodes and inserting no others) is strongly connected if and only if any node can be reached from any other. Maximal strongly connected (M.S.C.) subgraphs are defined with respect to particular nodes. For a given node, it is the largest strongly connected subgraph that contains that node. It is unique for a given node.

When both $G_{SE}(z)$ and $G_{ES}(z)$ are not equal to zero, the two nodes S and E belong to the same strongly connected subgraph, since they are mutually reachable.

An essential node with respect to $G_{SE}(z)$ is defined to be one which is reachable from S and can reach E. Under the same conditions for $G_{SE}(z)$ and $G_{ES}(z)$ (i.e., $\neq 0$) it is true that any essential node w.r. to $G_{SE}(z)$ is essential w.r. to $G_{ES}(z)$. (S and E are always (by definition) essential w.r. to $G_{SE}(z)$). This is so since any essential node on the "forward" path from S to E is either on a "reverse" path which is known to exist ($G_{ES}(z)\neq 0$) and is therefore essential to $G_{ES}(z)$ or it can be linked first to E then to any one of the reverse paths to S.

Ramamoorthy uses these concepts and facts to develop means of analytically, i.e., by matrix manipulation, determining when a graph has structural flaws in the form of entrance but no exit, or redundant (i.e., not essential) nodes w.r. to S and E.

The criterion $G_{SE}(z)\neq 0$ can be determined by forming a "reachability" matrix (with Boolean elements) by logical operations on the rows and columns of the connector matrix.

The major structural theorem is that a graph is strongly connected if and only if $G_{ij}(z)\neq 0$ for all nodes i and j.

Loops can be detected through use of the reachability matrix. (It is well to note that the Boolean-element connector matrix can be used to achieve this: by examining the main diagonal of the powers of the connection matrix. For that matrix there is no need to go beyond the nth power of an nxn matrix).

The presence of strongly connected subgraph is made manifest by the presence of all 1's (Boolean) in the matrix formed by taking the logical conjunct of the reachability matrix and its transpose. A given directed graph can be partitioned into separate maximal strongly connected subgraphs, and these could conceivably serve as aids to determining isolated regions for testing.

As noted above, for program testing and for describing the operational sequence caused by data, this technique does not appear to be very useful, at least in the form which presently is employed to depict them. The reason for this is that all of the potential path segments must be used, and would be represented as arcs between the graph nodes. Hence while nodes may be connected

"topologically" there may be no logical or numerical way for them to be connected (as part of an execution sequence). It may be noted that the use of this technique in hardware circuitry where "hard" connections exist between points, has been described by W. Mayeda and C. V. Ramamoorthy [9]. Nonetheless, this technique is useful in eliminating flaws, as noted before, and it probably will have use in testing of programs in an interactive mode. It would be necessary in that case for the user to employ the numerical values of the program variables which result during a particular execution sequence, so as to direct the computation along his chosen paths. But his choice of these paths could be materially aided by the isolation of maximally strongly connected subgraphs. This would be the case even if the arcs of the directed graph represent only potential program subpaths.

It has been suggested by M. Lipow [10], that the mathematical subfield of Lattice Theory may have application in the analysis of programs. In particular, it is proposed that the number of test cases required for exhaustive testing can be deteremined by application of theorem due to R. P. Dilworth [11].

In the suggested application the entities which have been defined earlier as segments serve as nodes of a directed graph, while the arcs of the graphs represent transfers between segments.

Using the order relation ($x \geq y$) between two elements, x and y, to mean y can be reached from x as defined in the earlier discussion; a program can be represented as a partially ordered set. Elements which are comparable can be put into chains linking together those that stand in the "forward reaching" relation. Mutually incomparable elements can be considered to comprise what is called an antichain. Execution sequences form chains from an initial node (starting segment) to the end node (halt segment). Dilworth showed that the smallest number of disjoint chains in a partially ordered set is equal to the largest number of mutually incomparable elements of the set. Lipow showed that the requirement that chains be disjoint, can be altered. He showed that the theorem is true if the chains are all maximal chains (i.e., like logical paths in software).

This fact, as with the Ramamoorthy (et.al.) results, is promising, but they cannot be applied as a direct measure of the number of realizable logical paths. The reason is as before stated, there is no way of knowing what potential paths actually are linked together until input data is employed to produce execution sequences.

B. Analytical Background for Generation of Random Numbers

1. Introductory Comments

Described below are the specifications for a program or subroutine which will provide random numbers for use as input variables to programs under test. All variables are confined to a finite range so that truncation is required in order to use distributions such as the normal (Gaussian) or exponential, which have infinite ranges. In order to fit any distribution to the finite range it is necessary to do some initial processing in order that those computer programs which have been developed as standard commercially available routines can be used. The analysis required is described here. Appendix I contains a description of the program (in Section II, Test Case Preprocessor).

IMSL (International Mathematical and Statistical Libraries) subroutines exist for many probability laws including the beta, uniform, and normal laws discussed here. In order to develop numbers from an arbitrary range, meeting other requirements with respect to shape and moments it is necessary to develop relations which will preprocess and postprocess the data entering into or coming from these subroutines.

In the following sections such processing as is required for three distributions is given: the beta distribution, triangular distribution and the truncated normal.

2. Beta Distribution

The routine for generating beta-distributed variables which is included in the IMSL package requires as input two parameters p and q which are required to be multiples of 1/2. The probability density function in terms of these parameters is given by the formula

$$F(x) = \frac{\Gamma(p+q)}{\Gamma(p)\ \Gamma(q)}\ x^{p-1}\ (1-x)^{q-1} \qquad (1)$$

on the interval $0 \leq x \leq 1$, and $F(x)=0$ elsewhere.

A variable linearly related to the beta variable through the linear transformation

$$u = L + Rx$$

is also beta distributed. This is a convenient form to start with, since u can be assigned to an arbitrary interval by specifying the lower limit L, and the range R.

The following relations between the variables hold

$$E(u) = L+RE(x) \tag{2}$$

$$\sigma_u^2 = R^2\sigma_x^2 \tag{3}$$

$$M = L+Rx_m \tag{4}$$

where E is the expectation operator (or average), σ_x^2 is the variance of x, M is the mode (location of the most likely u value) and x_m is the mode of x.

The relations (2), (3), and (4) can be solved to obtain the mean, variance, and mode of the standardized beta variable. The mean and variance of this variable whose density is given by (1) are

$$E(x) = \frac{p}{p+q} \tag{5}$$

$$\sigma_x^2 = \frac{pq}{(p+q)^2(p+q+1)} \tag{6}$$

The mode is

$$x_m = \frac{p-1}{p+q-2} \tag{7}$$

One of the difficulties with using the beta distribution is that arbitrary assignments to the mean, variance and mode cannot be made. Fortunately, there is, for this distribution, a wealth of experience developed (in the early 1960's) during the analytical background for the PERT technique.

Some of the early workers in PERT used two approximations that, while limiting the family somewhat, served well in the sense of producing realistic results: the standard deviation, in all cases, was chosen to be 1/6 of the range (R), and the average was taken to be 1/6 (L+4M+U), where L and U (U=L+R) are the lower and upper limits of the variable, and M is the "most likely" value.

Recapping, the process can be carried to the point of determining the parameters p and q by selecting L, R, and M, computing E(u) as E(u)=1/6 (L+4M+U), computing σ^2_u by taking 1/36 of the squared range (R^2) and solving equations (2) and (3) for E(x) and σ^2_x (=1/36).

It is necessary to develop a solution to equations (5) and (6). To do so m_1 is used to replace E(x) and m_2 is used to replace $\sigma_x^{\;2}$, then

$$m_1 = \frac{p}{p+q}$$

$$m_2 = \frac{pq}{(p+q)^2(p+q+1)}$$

Put u=p+q and q=u-p

then

$$m_1 = \frac{p}{u}$$

$$m_2 = \frac{p(u-p)}{u^2(u+1)}$$

and

$$m_2 = \frac{um_1(u-um_1)}{u^2(u+1)} = \frac{m_1(1-m_1)}{u+1}$$

so

$$u = \frac{m_1(1-m_1)}{m_2} - 1$$

hence $$p = \frac{m_1}{m_2}\ (m_1(1-m_1)-m_2) \tag{8}$$

and $$q = \frac{(1-m_1)}{m_2}\ (m_1(1-m_1)-m_2) \tag{9}$$

This general solution can be specialized in the present case, since m_2=1/36

$$p_s = m_1\ (36m_1(1-m_1)-1) \tag{8a}$$

and

$$q_s = (1-m_1)\ (36m_1(1-m_1)-1) \tag{9a}$$

Since GGBET, the IMSL routine which provides random numbers under a beta probability law, requires p and q to be multiples of 1/2 the nearest lattice point to the computed (p_s, q_s) point should be taken.

3. Triangular Distribution

The general form for the triangular density is given in Figure 3

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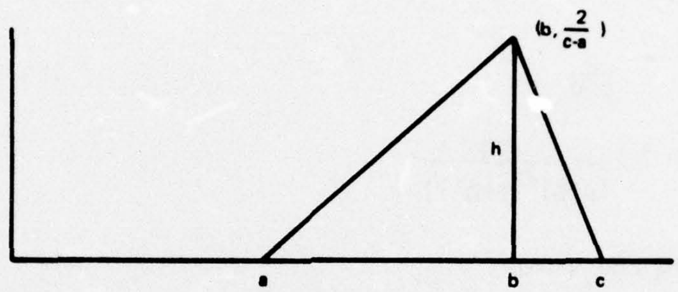

Figure 3. Triangular Density

For a density, the area must be 1, so that

$$1/2\,(c-a)\,h = 1$$

$$\text{or} \quad h = \frac{2}{c-a}$$

In general,

$$\text{Prob}\,(t \le z) = \int_a^z f(x)\,dx \qquad a \le z \le c$$

and in particular

$$\text{For } z \le b \quad = \int_a^z \frac{2}{c-a}\left(\frac{x-a}{b-a}\right)dx \qquad a \le z \le b$$

$$= \frac{2}{c-a} \cdot \frac{1}{b-a} \cdot \frac{(z-a)^2}{2} \qquad a \le z \le b$$

and for z>b

$$\text{Prob}\,(t \le z) = \frac{b-a}{c-a} + \int_a^z \frac{2}{c-a}\,\frac{c-x}{c-b}\,dx$$

$$= \frac{b-a}{c-a} + \frac{2}{c-a}\,\frac{1}{c-b}\left[\frac{(c-b)^2}{2} - \frac{(c-z)^2}{2}\right]$$

Simplifying,

$$F(z) = \frac{1}{c-a} \frac{(z-a)^2}{b-a} \quad a \le z \le b$$

$$= 1- \frac{(c-z)^2}{(c-b)\ (c-a)} \qquad b \le z \le c$$

Using (F(z) as the function relating the variables z and u, that is, for

$$u=F(z)$$

it is known that generally

$$p_u du = p_z d_z$$

where p_u and p_z are densities for the variables at corresponding values, and

$$p_z = \frac{dF(z)}{dz}$$

$$p_u = \frac{p_z}{\frac{du}{dz}} = \frac{p_z}{p_z} = 1$$

so that u is uniform on F(a)=0 to F(b)=1.

Since F(z) is monotone, its inverse function exists and the variables u and z can be related through the inverse,

$$z=F^{-1}(u).$$

The inverse function is given by

$$z = a+ \sqrt{(c-a)\ (b-a)u} \qquad \text{for } 0 \le u \le \frac{b-a}{c-a} \tag{10}$$

$$= c- \sqrt{(c-b)\ (c-a)\ (1-u)} \quad \text{for } \frac{b-a}{c-a} \le u \le 1. \tag{10a}$$

In operation u is chosen on the range 0 to 1, tested as to whether it is less or greater than $\frac{b-a}{c-a}$ and, depending on the result one or the other of the formulas is used to determine the Z variable, which is a triangularly distributed variable.

4. Truncated Normal Distribution

As a result of the operation of the GGNOR, the IMSL subroutine for generating numbers used a normal probability law, N normal variables are produced, each of which has a mean value of zero and a standard deviation of unity.

The usual application is to convert the standard normal variable into one with a mean m and a standard deviation σ. This is done by multiplying the standard normal variable, x_n, by σ and adding m, that is

$$z = \sigma x_n + m$$

is a normal variate with mean m and standard deviation of σ

For present purposes, it is desired that a truncated normal variable be produced. This can be done by specifying any interval along the real number line; however, for applications which are made in this study, the interval has the mean m as its midpoint. Thus, intervals of the form $(m-k\sigma, m+k\sigma)$, where k is any positive number, are employed.

This trunctated distribution is accomplished easily: since the program for generating the normal variables exists, it is somewhat inefficient, but mathematically correct to throw out those which are not in the range of the interval. In other words, the user takes the normal variables as they come, and eliminates those outside the range $(m-k\sigma, m+k\sigma)$. Usually, k will be chosen so that only a relative small proportion of the generated numbers are not used: for k=1.96, about 1 in 20 numbers are not used; and, for k=2.58, 1 in 100 are not used.

As a sample computation, suppose that a normal variable truncated to the real number range [a,b] is desired. It is necessary then to also specify the "amount" of truncation by specifying k - as indicated above k=1.96 will produce a set in which about 19 out of 20 occur in the truncation interval. For the analysis k is left open but would have to be specified as input data by the user. Since the normal is symmetric and, by the choice previously cited, the truncation interval is symmetric about the mean, then in terms formerly described

$$\frac{a+b}{2} = m$$

and

$$\sigma = \frac{b-a}{2k}.$$

Thus for a, b and k input, m and σ are produced. These are used to modify the subroutine-generated standard normal variables to the variate z previously

defined. The random numbers are further subjected to the truncation limits and eliminated or retained.

C. Program Testing System (Overview)

The preliminary processing of the input data by PTS provides a set of FORTRAN-conformable data elements, maintaining the format which the program under test requires of the data.

In order to collect the necessary data from a selected computer program, it is necessary to "instrument" the program by insertion of transparent instructions which can collect information relevant to the operation of the program. The backbone instrumentation is provided by a McDonnell Douglas Astronautics Company-developed program called PET. This is described as part of Appendix I, which discusses the special program PTS (Program Testing System) developed for this study. The preprocessor of this program automatically generates a FORTRAN program to prepare data in the format required for the execution of the particular program under test.

An additional augmentation of the basic system consists in a segment post-processor. This develops program segments from the listing, and subsequent to the running of a set of tests, prints tables describing the relative frequency of use of each segment.

The first post-processor report contains the FORTRAN source listing, followed by statement numbers which are assigned by PTS. Each executable FORTRAN statement is assigned a number. A logical IF statement is assigned two numbers, one for the IF portion, and one for the true branch, of the IF. This report allows the user to correlate the program segments with the actual source statements.

The second report describes the FORTRAN segments as defined by PTS. For each test case a cumulative number of times each segment was executed is shown. At the end of the report, the percentage of the segments that were executed for each case are printed.

The third PTS post-processor report consists in a list of the segments and the percentage of the cases that executed each segment. For instance, if a segment is exercised in 13 of 14 test cases, 93.33 would be printed.

The final report in the series is a summary of the segments that were not executed.

III DETECTION RATE MODELS
(PRELIMINARY)

Described in this section, in a preliminary way, are three detection rate models, two of these were initiated prior to the period of the contract, but were developed in directions which proved useful to the problem of estimation of the number of logical paths. That technique is illustrated in the next section under a description of the analysis of programs when driven by random numbers. A third model was developed at MDAC during the contract period and while the results have been presented to a national meeting of a professional society, there were no formal proceedings of that meeting published, and so the description is presented here. It is presented in an overview form here, and in a more detailed way in Section V which provides a comprehensive description of all three models.

In order to present the material in a manner which conforms to the earlier published work, all models are described as if they were failure or detection rate models with time as the independent variable. In the applications of these models to the major estimation problem described later, the independent variable is the trial number which "runs" like time, and detection corresponds to the finding of a segment or path not formerly exercised. The random nature of the input data makes each selection an "attempt" to find a new branch, where in the original context each unit of time corresponded to an "attempt" to detect a software error or anomaly.

A. De-Eutrophication Model

The assumption of a uniform detection rate over a program's development period is unrealistic. But when viewed at a more microscopic level, uniformity may have its place: over periods of time between the detection of successive errors, the assumption of uniformity merely interprets the system as being one where any remnant error (or unexercised path in the current context) can occur at any time. The basic model describing the detection of software failures was proposed by Z. Jelinski and P.B. Moranda [12] is indicated in Figure 4. The

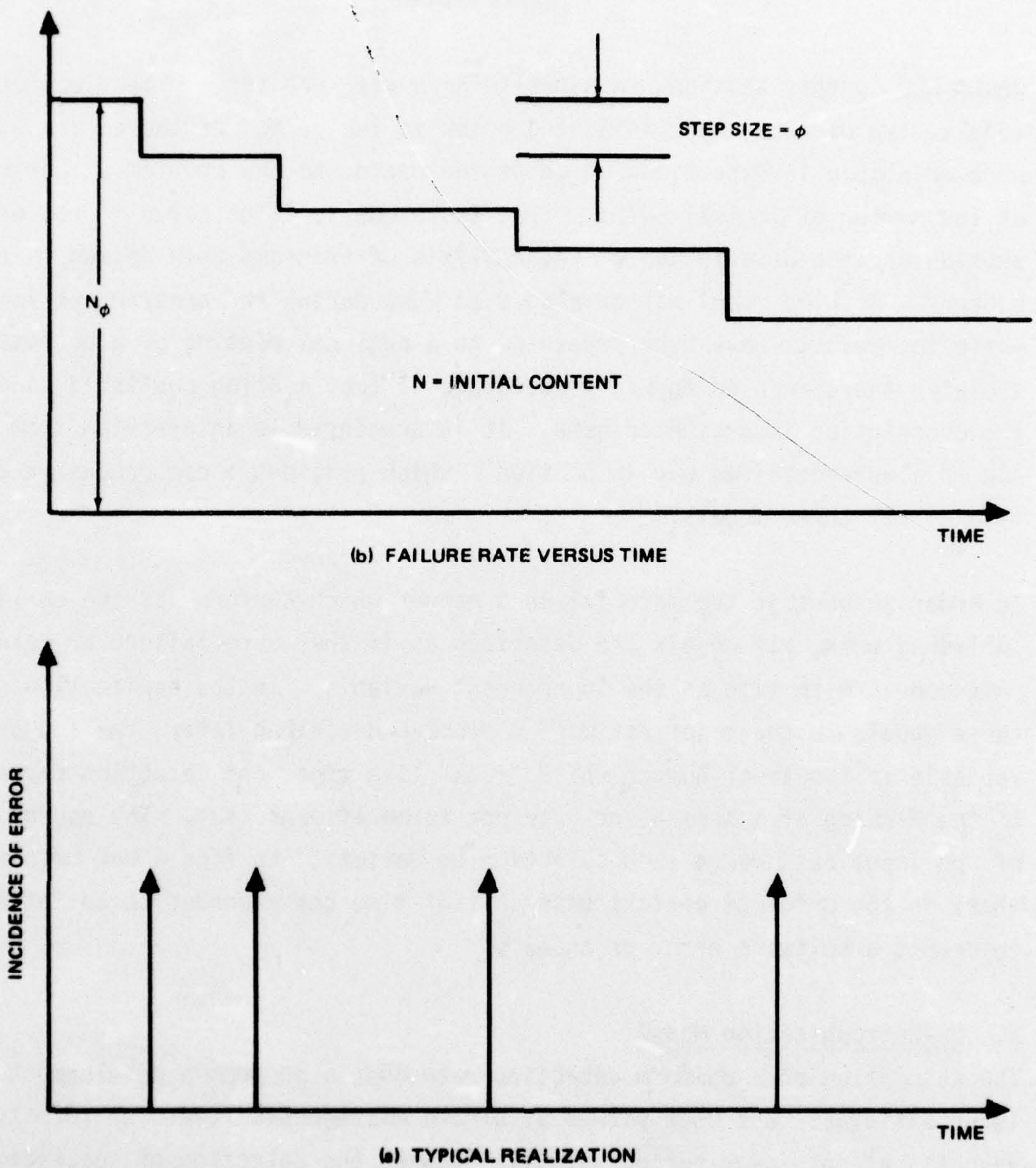

Figure 4. De-Eutrophication Process and Its Realization

detection rate at any time is assumed to be proportional to the current error content of the tested program. The initial error content is then denoted by N, and the proportionality constant is denoted by ϕ; the failure rate drops to $(N-1)\phi$ after the first error is detected, and so forth. The step size represents one "errors worth" of contribution to the total.

A typical realization of such a process is depicted in the lower portion of the figure, the increasing time between errors is purposely indicated by the spacing. The term "de-eutrophication" would seem to be appropriate to describe such a random process. The data comprising the observables are the times between adjacent errors. These are denoted by $X_1, X_2, \ldots, X_n$.

B. Geometric De-Eutrophication Process

While the basic model presented in the preceding section of this report may have much appeal, the data obtained in real applications may not fit the underlying assumptions. There are those who believe that there are not a finite number of errors in a large real-time program: certainly this is so if there is an attempt to mirror in software all of the continuum of eventualities which occur in complex dynamic situations. Also, the assumption that all errors have the same likelihood of detection is sometimes an imperfect rendition of the real situation.

In a variation of the basic model, both of these are to a degree alleviated. In this variation, proposed by Moranda [13], the step representing the decrease in failure rate between adjacent intervals, (which are defined, as before, by the occurrence or detection of an error), is taken to be a geometrically varying amount. This is represented in Figure 5.

Here again, the times are random variables and are mutually statistically independent. The observables for use in the analysis are, as before, the time separation between adjacent errors.

C. Hybrid Geometric Process

This model was described by Moranda [14] as a candidate for depicting the initial segment of hardware system testing. It covers the burn-in and steady state interval of time. The model also has applicability to the software process,

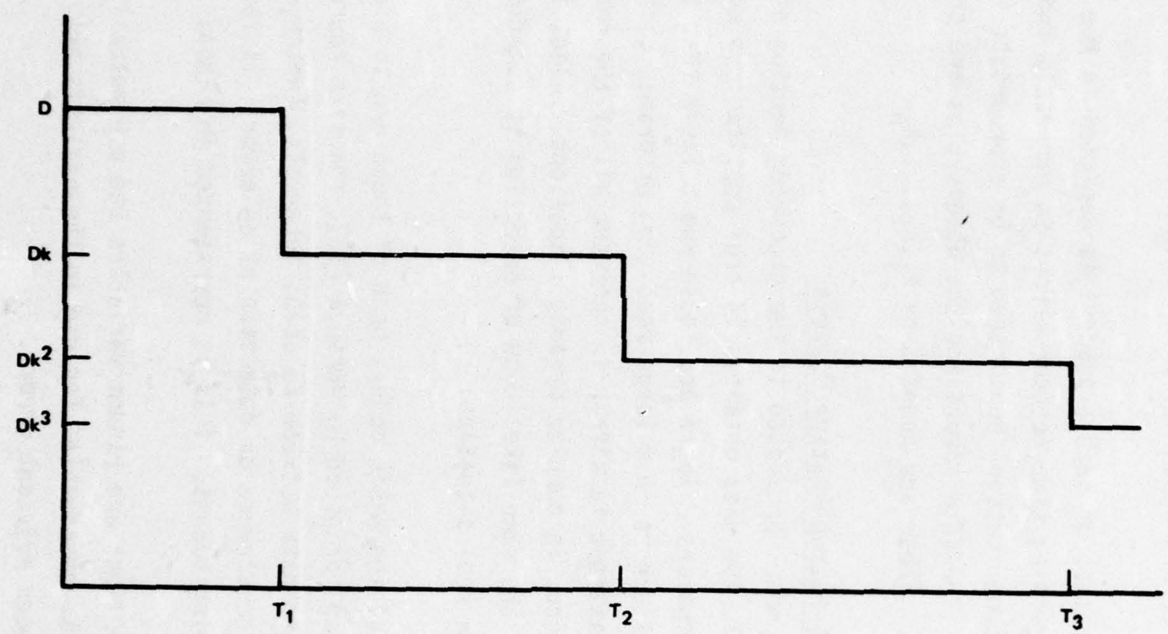

Figure 5. Geometric De-Eutrophication Process

producing estimates of the eventual MTTF, or in the more relevant context, the average number of trials to uncover a new segment.

The hybrid or composite model formed from the Geometric De-Eutrophication Model and a pure Poisson model, is depicted in Figure 6. The three parameters are: D, the initial term of the geometric progression; k, the ratio between successive terms; and θ, the parameter of the Poisson process.

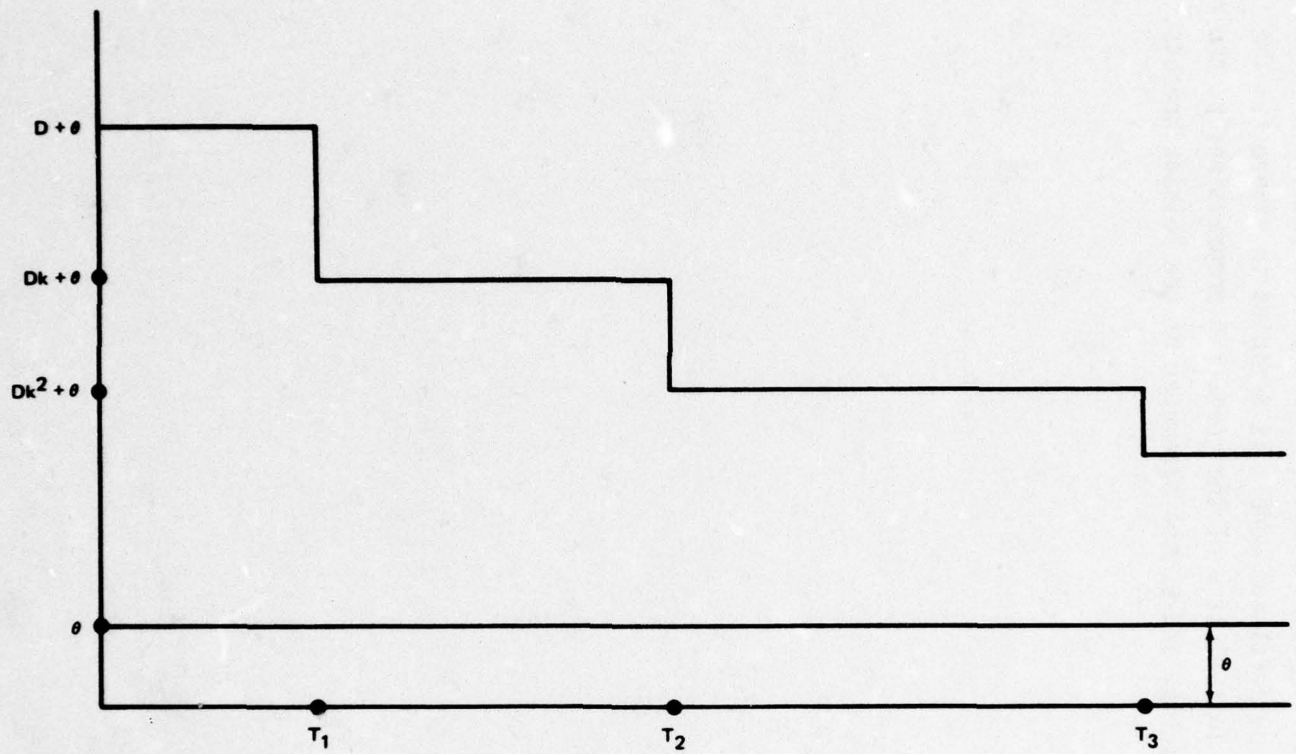

Figure 6. Hybrid Geometric Process

IV ANALYSIS OF PROGRAMS

With the brief background provided in the preceding sections, a review of the results obtained in the application of the techniques to three programs can be described.

These programs are each driven initially by random number generators, subsequently each is analyzed as to the requirements on the input data for driving some of the still unexercised program segments. In some cases there are segments which cannot be exercised; these are identified.

The programs are discussed in the order in which they were studied. As a consequence the presentations are not the same. The first program is discussed at greatest length, although the most significant techniques are discussed in the description of the second program and its test results.

A. Lehmer Root-Solver

1. Description of the Root-Solving Method

The first program analyzed is a general purpose polynomial root solver based on the so-called Lehmer method.

This method, described by D. H. Lehmer in JACM [15], is basically a search process consisting of sequences of overlapping circles and annuli with decreasing radii in the complex plane. While the procedure is not a topic vital to the understanding to the testing process, it is well to describe it in some detail so that the discussion of various points which are made here will be clearer.

For a polynomial $f(z)$, with complex coefficients, rings of the form

$$R < |z| < 2R$$

are formed. If, as it may be assumed, the polynomial does not have zero as a

root, $f(0) \neq 0$, the process of doubling or halfing the radius will eventually result in an annulus which has no zeros inside the inner ring and one or more inside the outer ring. A basic algorithm determines when a given circle contains one or more roots of the polynomial. The so-found and conditioned ring can be covered by 8 overlapping circles, each with radius $\frac{5}{6}R$, and with centers at $\frac{5}{3}R(\exp[\frac{2\pi ik}{8}])$ for $k = 0, 1, 2, \ldots, 7$. One of these must contain at least one root. If taken in sequence, the first one which contains a root, as determined by the test algorithm mentioned above and described more fully later, is subjected to further examination. Because of the overlap, this root (or roots) may fall outside of the original annulus; if so, the procedure of successive encircling is carried out on that root until the value of the root is found, and then the procedure is restarted with the original sequence of circles operating on the reduced polynomial. (Since all roots are finite, the process of diverging away from the main focus of the search cannot continue without end).

With the center of the circle which is (first) known to contain a root, a new annulus of the form

$$R_1 < |z-\alpha_1| < 2R_1$$

(actually, $R_1 = \frac{5}{6} R \, 2^{-\theta}$ where θ is a positive integer) is used. This annulus is covered by 8 circles of smaller radius and the first one containing a root is selected. In this way, a sequence of circles is constructed whose radii form a convergent geometric (null) sequence, and the root can be found to any desired accuracy.

Fundamental to the procedure is an algorithm which establishes whether or not the interior of the circle

$$|z-c| = \rho$$

has a root of the polynomial equation $f(z)=0$.

By a simple linear transformation, the given circle can be transformed to the unit circle, and, with the notation $g(z)=f(\rho z+c)$, the root location decision process can be carried out starting with the unit circle and using $g(z)=0$ as the canonical polynomial equation.

The linear combination

$$T(g(z)) = \bar{a}_0\, g(z) - a_n z^n \bar{g}\,(\bar{z}^{-1})$$

is the fundamental device for producing the answer to the question of whether the unit circle contains a root of g.

T(g(z)) is a polynomial which, by construction, has no z^n term, and is therefor of degree lower than n. The constant term of T(g(z)) is

$$T(g(0)) = |a_0|^2 - |a_n|^2$$

If this is differnt from zero then T(g(z)) (has no zero root) is a polynomial on which the linear combination T can be applied, as before.

Such a process can be continued through the sequence T(g), $T^2(g)$, ..., $T^k(g)$ where $T^k(g(0))$ is identically zero. This is so since the degree of the polynomials in the sequence decreases under each application of T.

With this as background, the following fundamental theorem can be stated:

> Let g (z) have no zero on the unit circle. If, for some $h>0$, $T^h(g(0))<0$, then g has at least one root inside of the unit circle. On the other hand, if $T^i(g(0))>0$ for all $0\leq i<k$ and $T^k(g(0))$ is constant, then no root of g lies inside the unit circle.

This theorem is proved by the use of four lemmas, two of which are the Cauchy Integral Theorem, and the Theorem of Rouche; the other two have to de exclusively with polynomials and are interesting in their own right.

The first of these is

> **Lemma:** For P and Q polynomials such that $|P(z)| < |Q(z)|$ on the unit circle, then Q and P+Q have the same number of roots inside the unit circle.

The second lemma is:

Lemma: Let $\emptyset$ be a polynomial of degree d with no root on the unit circle and m roots inside. If $T(\emptyset(0))\neq 0$ then $T(\emptyset)$ has no root on the unit circle and has m or d-m roots inside according as $T(\emptyset(0))$ is positive or negative.

A useful fact is developed in a second theorem:

Theorem. The previous theorem is true even if g(z) has a zero on the unit circle.

An example from Lehmer's paper can be used to clarify the procedure

For

$$g(z) = -8z^3 - 14z^2 + 3z + 9$$

the function $g^*(z) = z^n\overline{g}(\overline{z}^{-1})$ is

$$g^*(z) = -8 - 14z + 3z^2 + 9z^3$$

so that

$$T(g(z)) = -6z^2 - 5z + 1$$

Hence

$$T(g(0)) > 0.$$

With reapplication of the procedure to the polynomial

$$g_1(z) = T(g(z)),$$

there results the function

$$g^*_1(z) = -6-5z + z^2$$

and

$$T^2(g) = -35z-35$$

Hence

$$T^2(g(0)) < 0$$

and so by the theorem g has a root inside the unit circle.

To show the use of the annuli, the original polynomial is transformed by replacing z by $\frac{z}{2}$. Then, after eliminating the common factor:

$$g'(z) = -2z^3 - 7z^2 + 3z + 18.$$

Application of the T-transformation results in the sequence

$$T(g'(0)) = 320$$

$$T^2(g'(0)) = 880$$

$$T^3(g'(0)) = 391$$

and $T^i(g'(0))$ are all positive and there are no roots of $g'(z)=0$ inside the unit circle. Thus, for the original polynomial, there are no roots inside the circle $|z|= \frac{1}{2}$. Thus, it can be stated that the original polynomial equation has one or more roots in the annulus

$$\frac{1}{2} < |z| < 1$$

but none inside

$$|z|= \frac{1}{2}$$

Actually, the roots of $g(z)=0$ are 3/4, -1, and -3/2, as can be verified by substitution.

2. Lehmer Program Description

The program which employs this algorithm is coded in FORTRAN, and consists of three major subroutines. The first, denoted LEHMER, maintains the main stream of the development, forming the circles and annuli on the complex plane and testing the criteria for continuing the search. This routine uses the other two subroutines for the repetitive operations: the first, TOOO, is used to compute the coefficients of the linear combination $T(g(z))$, and evaluate the simpler combinations, such as $T(g(0))$; the second, SOOO, is essentially a polynomial evaluation routine, taking the coefficients of the polynomial $f(z)$ (or $T(g(z))$) and evaluating them at particular points.

The coding for subroutine TOOO is listed in a later figure (see Figure 7). The description of this program in the computing manual is given below.

<u>LEHMER</u>

Double Precision Polynomial Root Finder SUBROUTINE

a. <u>Description</u> - This subroutine subprogram finds all the real or complex roots of a polynomial with real or complex coefficients.

b. <u>Use</u> - CALL LEHMER (ar, ai, n, rr, ri, er, ei, r, z, i, br, bi, cr, ci, qr, qi, dr, di, tq) where:

n — is an INTEGER variable. This variable is the degree of the polynomial.

ar — is a DOUBLE PRECISION array dimensioned n+1. The elements of this array are the real parts of the polynomial coefficients beginning with the constant term.

ai — is a DOUBLE PRECISION array dimensioned n+1. The elements of this array are the imaginary parts of the polynomial coefficients beginning with the constant term.

r — is a DOUBLE PRECISION variable. If the magnitude of the evaluation check is less then r, the root is acceptable. If r is not in the range $10^{-13} \leq r \leq 10^{-5}$, then is used.

z — is a DOUBLE PRECISION variable. If the magnitude of the difference between two consecutive approximations to the root is less than z, then the root is acceptable. If z is not in the range $10^{15} \leq z \leq 10^{-7}$, then 10^{-9} is used.

i — is an INTEGER variable. If i=0, complex conjugates of roots are found when all of the polynomial coefficients are real. If i=1 then no complex conjugates are found.

br, bi, cr, ci, dr, di, qr, qi, tq — are DOUBLE PRECISION arrays dimensioned n+1. These arrays are used as working storage.

rr — is a DOUBLE PRECISION ARRAY DIMENSIONED n. The elements of this array are the real parts of the complex roots.

ri is a DOUBLE PRECISION array dimensioned n. The elements of this array are the imaginary parts of the complex roots.

er is a DOUBLE PRECISION array dimensioned n. The k^{th} element of this array is the real part of the polynomial evaluated at the k^{th} root.

ei is a DOUBLE PRECISION array dimensioned n. The k^{th} element of this array is the imaginary part of the polynomial evaluated at the k^{th} root.

c. Support Level - Supported by Programming Systems and Support Branch of Information Processing Systems.

d. Language Used - FORTRAN.

e. Availability - On FORTRAN library.

f. Extent - 6414_8 words.

g. Timing - Not available.

There are three programmer-specified options: an evaluation check, in which f(z) evaluated at the root, z_i, is subject to the test $|f(z_i)| < r$; a convergence circle check in which successive approximations to the root are compared to an assigned value zeta; and a short-cut which employs the conjugate of a found root in case of a real polynomial.

3. Analysis of PTT-Segment Data

This program, when instrumented with the Program Testing Translator results in 406 PTT-segments, which include segments which are terminated not only by branching statements, but by any label; thus, a GO TO (LABEL) would be the terminating instruction in a PTT-segment, while it would not be in the segments as defined earlier. These PTT-segments are useful as they stand, since they provide usage information directly, i.e., without a second processing of the instrumented program. In the later more detailed work, these PTT-segments were used to develop the longer segments for the TOOO and SOOO subroutines. This

resulting composition resulted in 94 segments for the T000 subroutine, made up from 195 PTT-segments. The S000 subroutine was composed into 23 segments from 61 PTT-segments.

Much of the early work, however, was done using the PTT-segments since the statistics which were developed automatically were referenced to those segments. In the initial runs triangular, uniform, and beta distributions (defined in Section II) were used.

Described first is a typical result. Polynomials of degree 4 were formed by random selection of their coefficients from the interval $[10^{-14}, 10^{14}]$ using a triangular probability density function; the resulting number for each coefficient was assigned a positive or negative sign on a 50-50 random basis. A batch of 10 polynomials so formed were employed in this typical run.

The first case (of the 10 cases), with the randomly selected coefficients

$$
\begin{aligned}
a_0 &= 1.427044 \times 10^{-4} & b_0 &= -6.719286 \times 10^{-6} \\
a_1 &= 7.940723 \times 10^{-3} & b_1 &= -7.710267 \times 10^{0} \\
a_2 &= 4.145428 \times 10^{-6} & b_2 &= -7.922906 \times 10^{2} \\
a_3 &= -2.621762 \times 10^{-6} & b_3 &= -6.921374 \times 10^{-3} \\
a_4 &= -1.066273 \times 10^{2} & b_4 &= -5.723998 \times 10^{+2}
\end{aligned}
$$

exercised 204 of the 406 PTT-segments.

As a matter of interest, the resultant polynomial was found to have the roots

$$
\begin{aligned}
r_1 &= -8.67 \times 10^{-8} + j\,(1.85 \times 10^{-6}) \\
r_2 &= -9.67 \times 10^{-2} + j\,(-1.308 \times 10^{-4}) \\
r_3 &= -5.87 \times 10^{-2} + j\,(1.16) \\
r_4 &= .155 + j\,(-1.16)
\end{aligned}
$$

The degree of accuracy of these roots can be measured by the evaluation of the polynomial at the root (i.e., $f(r_1)$, $f(r_2)$, etc.) These are

$$
\begin{aligned}
f(r_1) &= -3.9 \times 10^{-9} + j\,(-1.4 \times 10^{-8}) \\
f(r_2) &= -2.0 \times 10^{-9} + j\,(-1.5 \times 10^{-8}) \\
f(r_3) &= -2.0 \times 10^{-8} + j\,(-1.0 \times 10^{-6}) \\
f(r_4) &= 3.2 \times 10^{-7} + j\,(-9.6 \times 10^{-7})
\end{aligned}
$$

These evaluations are useful for the typical user, and it is interesting to note that in 3 of the first 10 cases run (the batch which is the subject of the discussion), one or more of the roots were not located with what at first look, would be judged as satisfactory accuracy. As an example, one of the cases had a value of $|f(r)|$ of 10^9. On inspection, the coefficients in this case had magnitudes which with one exception, vary from large to very large (10^5, 10, 10^2, 10^7). Moreover, one root has a magnitude of 10^7, and some values in the evaluation of the polynomial at that root would be as large as 10^{28} (the Z^3 term has coefficient of "about" 10^7). Viewed in this light the accuracy is less suspect and the program does well under the stress imposed by the large numbers used.

As noted before, the interest is not in the correctness of the programs being tested; however, the above discussion does describe some of the details which are descriptive of the program.

a. <u>Aggregate Effects</u>

A straightforward table of the cases shows the following sets of numbers exercised against the case number

Table I

Triangular Distribution Results

Case	No. of Segments Exercised
1	204
2	199
3	202
4	205
5	200
6	197
7	197
8	201
9	196
10	208

The number not exercised by any of the test cases totaled 183. Thus, the program was exercised by at least one of the test cases in 223 out of 406, or a percentage of 54.9. Because the smallest number in the above table represents 49.2%, there is much overlap in the testing.

For purposes of illustrating the analysis described later, the following data is useful. It lists the trial number versus number of "new" PTT-segments exercised. The initial number if 204 as indicated in the above table. Even though there are fewer total in case 2, there are some branches (PTT-segments) taken during this execution which did not occur in the first case. The actual number is 6. The third case exercises 1 PTT-segment not exercised by either case 1 or 2. The fourth exercises 5 not previously exercised. The fifth exercises 1 new segment; 1 new segment is found in case 6; 3 in case 7; 1 in case 8; none in case 9, and 1 in case 10.

Data of this type are useful in analysis of the economic stop point. In the present instance, there is no analysis necessary and no reason to stop testing since, in all but one case there is a non-zero yield in the new cases found.

b. Distributional Effects

At this point, it is helpful to discuss the effects which different distributions have on the results. For this purpose, runs of size 10 which were made with each of the three distributions (uniform, beta, and triangular on 10^{14} to 10^{14} with random signs) can be used.

Intuitively, the beta and triangular distributions should furnish almost the same in-the-large results since they are shaped nearly the same. This indeed is found to be the case. None of the cases in the "beta-batch" exercised any segments not already included in the triangular. This is probably within the "noise" of samples of this size: ten additional cases of either distribution would probably show a similar comparison.

On the other hand, the triangular (and beta) differ considerably in shape from the uniform, and it is expected that there would be a difference in the results. This turns out to be the case, with 10 segments exercised by the 10-size "uniform-batch" not exercised by the same sized "triangular batch", and, for completeness, 2 in the latter batch not in the former.

The results of the ten cases for the uniform are summarized below in Table II.

Table II

Uniform Distribution Sample

Case	No. of Segments Exercised
1	213
2	196
3	202
4	187
5	197
6	202
7	188
8	199
9	197
10	179

The number of segments exercised are nearly the same as those for the triangular distribution as shown in Table II. As noted, not only do the numbers correspond,the actual paths correspond. Of interest are cases 1 and 10; the latter has the fewest actual number of segments exercised, while the former exceeds (by five) the largest of the number found in any uniform case.

The section of code which is caused to be entered by the data of this test case is governed by a test of the magnitude of the coefficient of the highest degree of a "working" polynomial. The actual test is of the form

$$\text{IF } (|\text{Im}(c_m)|-10^9) \text{ B,B,C}$$

where c_m is the mth coefficient of a "working" polynomial and the B-labelled instruction is the entry into the section of code. The imaginary part of the working polynomial coefficient is

$$\text{Im}(c_m) = (\text{Re}(a_1))(\text{Im}(a_m)) - (I_m(a_1))(\text{Re}(a_m))$$

But for m=n-1, the actual data shows

$$\mathrm{Re}(a_1) = 5.2 \times 10^{-9}$$
$$\mathrm{Im}(a_{n-1}) = 3/4 \times 10^{-12}$$
$$\mathrm{Im}(a_1) = -4.4 \times 10^{-13}$$
$$\mathrm{Re}(a_{n-1}) = 3.1 \times 10^{-13}$$

so that the branch B is taken for this data.

Under a triangular or beta distribution (with symmetry about the midpoint of the interval) the values 10^{-13} and 10^{-12} which occur in this uniform case would indeed be data freaks.

The 10th case is of interest because it exercises only 179 PTT-segments. This is caused by the fact that some of the coefficients of the test case are very large (10^5, 10^5, 10^9, 10^6 for the first four coefficients). The test on the working polynomial (which always contains as one factor in each of four multiplications, either 8.9×10^{11} or -2.1×10^5) is directed away from a particular area of code, represented by 14 PTT-segments.

These two cases illustrate the benefit of testing with the uniform distribution. It stresses the program in small samples in a way which could only be achieved by large samples from the triangular or beta distribution.

It is relevant and necessary in a complete description, to point out that in programs of the type studied here, there are a great many CALLS to subroutines with data generated by intermediate computations. There are, for example, 287 calls (in Case 10 of the uniform distribution) of the major subroutine T000. Thus, for a single random number (set) on the input, there is in this case a great proliferation of numbers with which to test the segments of the subroutines.

To show the essential difference here, a coarse look can be made at the segment numbers, which are, even for this program which has a very large number of GOTO instructions, correlated with the actual execution sequence. It is noted that the largest number of times any of the first 50 listed PTT-segments is exercised is 9, and this occurred in a DO-loop which was entered three times and iterated 3 times. The first large usage number (58) occurred at the instruction which called T000; the next significantly

large number occurred at an instruction which has 8 different (i.e., "gone to") entry paths. The point to be made is, that "early" in a program the distributional effects may be more pronounced than in later portions or in repeatedly called subroutines.

The degree of the buildup is clearly shown by noting that while there is only one entry to LEHMER, in the case at hand, the entry to T000 is made 308 times, and S000 is entered 406 times.

It is along the same line, that the following observation, which while obvious, is of practical value. It is a fact that the large sample runs by any distribution over the same range on the input domain will produce the "same" results (i.e., exercise, in total, the same program segment).

The purpose in using particular distributions is to reflect the "real" world and that was the original reason for choosing different distributions. This is of little relevance in the area of testing which is aimed at exercising the greatest number of program paths. On the other hand, it is well to note here, that particular distributions reflecting reality have use in developing a priori estimates of software reliability. This point is discussed in Section VI.

4. Analysis of Aggregated Segment Data

a. Random Input Data

It is instructive to follow insofar as is possible a single input data case through the program. This has been done to the extent possible and the results are presented below.

First, it is noted that the trace through the entire program is extremely tedious and a more limited investigation is made, that being a trace through the subroutine T000. This is done with segments as defined in Section II, and requires chaining of some of the PTT-segments which were employed in the analysis above. The so-chained segments are listed in Table III and are obtained from the coding for the subroutine shown in Figure 7. A total of 94 segments are shown, and these are obtained from the 194 PTT-segments (and 155 instructions) in the T000 subroutine. The analysis shows that T000 is called 147 times by the LEHMER subroutine in Case I. The number

Table III

T000 Segments

T_1:	[78,273,274,275,276,277,278,279,280,281,282,283,284,285,286,287,383,384)
T_2:	[384,385,386,387,388,389,286,287,383,384)
T_3:	[384,390)
T_4:	[390,391,392,294)
T_5:	[390,288,289,290,291,292,293,393)
T_6:	[294,297,298,299)
T_7:	[294,295,296,305)
T_8:	[299,285,286,287,383,384)
T_9:	[299,300)
T_{10}:	[393,394)
T_{11}:	[393,402)
T_{12}:	[305,297,298,299)
T_{13}:	[305,314)
T_{14}:	[305,306)
T_{15}:	[300,303,304,285,286,287,383,384)
T_{16}:	[300,301,302,376,377]
T_{19}:	[394,395)
T_{20}:	[394,402)
T_{21}:	[402,344,345,346,347,410,411)
T_{22}:	[402,294)
T_{23}:	[314,315)
T_{24}:	[314,316,317,376,377]
T_{25}:	[306,376,377]
T_{26}:	[306,307,308,309,328)
T_{27}:	[306,312,313,308,309,328)
T_{28}:	[306,310,311,308,309,328)
T_{29}:	[395,396)
T_{30}:	[395,402)
T_{31}:	[396,397)
T_{32}:	[396,402)
T_{33}:	[411,412,413,414,415,345,346,347,410,411)
T_{34}:	[411,348)
T_{37}:	[315,320,321,322)
T_{38}:	[315,318,319,377]
T_{39}:	[328,329)
T_{40}:	[328,333,334,335,336,337,338,339,340,403,404)

T_{41}: [397,416,417,418,419,398,399)

T_{42}: [397,420,421,422,423,424,425,426,427,398,399)

T_{43}: [348,349,350)

T_{44}: [348,356,357)

T_{47}: [322,326,327,285,286,287,383,384)

T_{48}: [322,323)

T_{49}: [329,330)

T_{50}: [329,331,332,376,377]

T_{51}: [404,405,406,407,408,409,336,337,338,339,340,403,404)

T_{52}: [404,341,342)

T_{53}: [399,400)

T_{54}: [399,401)

T_{55}: [350,297,298,299)

T_{56}: [350,351)

T_{57}: [357,358)

T_{58_A}: [357,297,298,299)

T_{58_B}: [357,364,365)

T_{59}: [323,376,377]

T_{60}: [323,324,325,376,377]

T_{61}: [323,325,376,377]

T_{62}: [330,378)

T_{63}: [330,376,377]

T_{64}: [330,380,381,382,285,286,287,383,384)

T_{65}: [342,393)

T_{66}: [342,343,344,346,347,410,411)

T_{67}: [400,358)

T_{68}: [400,297,298,299,300)

T_{69}: [400,364,365)

T_{70}: [401,314)

T_{71}: [401,297,298,299,300)

T_{72}: [401,306)

T_{73}: [351,354,355,297,298,299)

T_{74}: [351,352,353,376,377]

T_{75}: [358,359)

T_{76}: [358,360,361,376,377]

T_{77}: [378,379)

T_{78}: [378,376,377]

T_{79}: [365,366,367,368,369,370,371)

T_{80}: [365,372)

T_{81}: [359,362,363)

T_{82}: [359,318,319,377]

T_{83}: [379,376,377]

T_{84}: [379,380,381,382,285,286,287,383,384)

T_{85}: [371,335,336,337,338,339,340,403,404)

T_{86}: [371,372)

T_{87}: [372,374,375,376,377]

T_{88}: [372,373)

T_{89}: [373,378)

T_{90}: [373,376,377]

T_{91}: [373,380,381,382,285,286,287,383,384)

T_{92}: [363,376,377]

T_{93}: [363,320,321,322)

```
      SUBROUTINE TOOO  (AREAL,QOOOFL,N,TAU,S,ALPHAR,ALPHAI,W,I1,I12,
     1BREAL,BIMAG,CREAL,CIMAG,QR   ,QIMAG,TR)
      DIMENSION AREAL(1),QOOOFL(1),BREAL(1),BIMAG(1),
     1CREAL(1),CIMAG(1),QR   (1),QIMAG(1),TR(1)
      DOUBLE PRECISION  AREAL, QOOOFL, BREAL, BIMAG, CREAL
      DOUBLE PRECISION  CIMAG  , QR     , QIMAG  , TR     , T
      DOUBLE PRECISION  S      , ALPHAR , ALPHAI , VREAL  , VIMAG
      DOUBLE PRECISION  T1     ,          TAU    , TK1    , W
      DOUBLE PRECISION  TK2    , CNORM
273   CALL OVERFL(KOOOFX)
274    GO TO(275,275)KOOOFX
275   M=N+1
276   T=S
277   LS=0
278   LT=0
279   LZ=0
280   MX=1
281   MY=1
282   MQ=0
283   IN=I1
284   KL=4
285    CALL SOOO  (AREAL,QOOOFL,N,1,T,BREAL,BIMAG,
     1ALPHAR,ALPHAI,VREAL,VIMAG,CREAL,CIMAG)
286   T1=(BREAL(1)**2+BIMAG(1)**2)-(BREAL(M)**2+BIMAG(M)**2)
287   GO TO 383
288   NL=M
289   CREAL(1)=BREAL(1)
290   CIMAG(1)=BIMAG(1)
291   CREAL(NL)=BREAL(M)
292   CIMAG(NL)=BIMAG(M)
293   GO TO 393
294   IF(DABS(T1)-TAU) 297,297,295
295   LS=0
296   GO TO 305
297   T=1.5D0*T
298   LS=LS+1
299   IF (LS-5) 285,300,300
300   IF (IN) 303,301,303
301   T=32.0D0/729.0D0*T
302   GO TO 376
303   T=10.0D0*T
304   GO TO 285
305   IF (T1) 314,297,306
306   GO TO (376,376,376,307,310,312),KL
307   KL=1
308   MQ=0
309   GO TO 328
310   KL=2
311   GO TO 303
312   KL=3
313   GO TO 308
314   IF (IN) 316,315,316
315    GO TO (320,320,320,320,320,318),KL
316    I2=1
317   GO TO 376
318    W=.5D0*T
319    GO TO 377
320   T=.5D0*T
321   MQ=MQ+1
322   IF (MQ-20) 326,326,323
323   GO TO (376,376,376,324,325,325),KL
324   T=32.D0*T
325    GO TO 376
326    KL=5
327   GO TO 285
328   IF (N-1) 329,329,333
329    IF (IN) 331,330,331
330    GO TO (378,376,380),KL
331    I2=2
332   GO TO 376
333   JJ=N+1
334   L=2
335   J=JJ
336   DO 339 I=1,JJ
337   CREAL(I)=(BREAL(1)*BREAL(I)+BIMAG(1)*BIMAG(I))-
     1(BREAL(JJ)*BREAL(J)+BIMAG(JJ)*BIMAG(J))
338   CIMAG(I)=(BREAL(1)*BIMAG(I)-BIMAG(1)*BREAL(I))-
     1(BIMAG(JJ)*BREAL(J)-BREAL(JJ)*BIMAG(J))
339   J=J-1
340   GO TO 403
341   NL=N+2-L
342   GO TO (393,343),MY
343   MY=1
344   GO TO 346
345   CONTINUE
346   TR(L)=(CREAL(1)**2+CIMAG(1)**2)-(CREAL(NL)**2-
     1CIMAG(NL)**2)
347   GO TO 410
348    IF (DABS(TR(L))-TAU) 349,349,356
349    LT=LT+1
350   IF (LT-5) 297,351,351
351    IF (IN) 354,352,354
352    T=32.0D0/729,0D0*T
353   GO TO 376
354    T=10.0D0*T
355   GO TO 297
356    LT=0
357   IF (TR(L)) 358,297,364
358    IF (IN) 360,359,360
359    GO TO (362,362,318),KL
360    I2=3
361   GO TO 376
362   LZ=LZ+1
363    IF (LZ-20) 320,320,376
364      LZ=0
365   IF(L-N) 366,372,372
366   DO 368 I=1,JJ
367   BREAL(I)=CREAL(I)
368   BIMAG(I)=CIMAG(I)
369   JJ=JJ-1
370   L=L+1
371   IF (L-N) 335,335,372
372   IF (IN) 374,373,374
373    GO TO (378,376,380),KL
374    I2=4
375   GO TO 376
376   W=T
377   RETURN
378   IF (ALPHAR) 376,379,376
379   IF (ALPHAI) 376,380,376
380   T=2.0D0*T
381   KL=6
382   GO TO 285
383   CALL OVERFL(KOOOFX)
384    GO TO(385,390),KOOOFX
385   DO 387 I=1,M
386   BREAL(I)=BREAL(I)*10,0D-10
387   BIMAG(I)=BIMAG(I)*10,0D-10
388   MX=2
389   GO TO 286
390   GO TO (288,391),MX
391   MX=1
392   GO TO 294
393   IF(DABS(CREAL(1))-10.0D-10) 394,394,402
394   IF(DABS(CIMAG(1))-10.0D-10) 395,395,402
395   IF (DABS(CREAL(NL))-10.0D-10) 396,396,402
396   IF (DABS(CIMAG(NL))-10.0D-10) 397,397,402
397   GO TO (416,416,416,420,420,420),KL
398   CNORM=(CREAL(1)**2+CIMAG**2)-(CREAL(NL)**2+CIMAG(NL)**2)
399   GO TO (400,400,400,401,401,401),KL
400    IF (CNORM) 358,297,364
401   IF (CNORM) 314,297,306
402   GO TO (345,345,345,294,294,294),KL
403   CALL OVERFL(KOOOFX)
404    GO TO(405,341),KOOOFX
405   DO 407 I=1,JJ
406   BREAL(I)=BREAL(I)*10.0D-10
407   BIMAG(I)=BIMAG(I)*10.0D-10
408   MY=2
409   GO TO 386
410   CALL OVERFL(KOOOFX)
411    GO TO(412,348),KOOOFX
412   DO 414 I=1,NL
413   CREAL(I)=CREAL(I)*10.0D-10
414   CIMAG(I)=CIMAG(I)*10.0D-10
415   GO TO 346
416   DO 418 I=1,NL
417   CREAL(I)=CREAL(I)*10.0D+10
418   CIMAG(I)=CIMAG(I)*10.0D+10
419   GO TO 398
420   DO 422 I=1,M
421   BREAL(I)=BREAL(I)*10.0D+10
422   BIMAG(I)=BIMAG(I)*10.0D+10
423   NL=M
424   CREAL(1)=BREAL(1)
425   CIMAG(1)=BIMAG(1)
426   CREAL(NL)=BREAL(M)
427   CIMAG(NL)=BIMAG(M)
428   GO TO 398
      END
```

Figure 7. TOOO Subroutine Coding

of times each segment is exercised is listed in Table IV. There is an apparent law of conservation: any end-node of a segment with say N counts must link to start-nodes which total N counts (if there are no other linkings to those nodes). Thus, T_{10} with 85 counts and end-node No. 394 links to T_{19} with 85 and T_{20} with zero. This process becomes complex because of loops which exist, many of which are not at all obvious. Thus, many of the segments have more than one (apparent) lead-in path, making the balancing process somewhat difficult. Thus, the 440 count for segment T_{11} and the 85 for segment T_{10} arise from two sources, 221 from T_5 and 304 from T_{65}.

It is not feasible to establish the program's execution sequence through this routine. To do so would require a trace from the start - that is, from the start in the LEHMER routine - through the 147 CALLS of this routine, and this is manifestly impractical, and very likely impossible from the aggregate data alone.

However, potential paths can be formed. In some cases, actual partial paths can be inferred, but, in other cases, no rescue is possible.

Figure 8 shows the paths which were exercised in Case 1 of the uniform sample. It presents a good status keeper for the test cases. Of the 93 segments which are included in TOOO, the case which was selected, exercised 50 segments (the figure 53.7% of segments agrees very well with the 52.46% figure for the PTT-segments; these should, of course, correlate well, but the closeness in actual values is fortuitous).

From the printout for the 10-size "uniform" - batch, the "union" of all segments exercised can be found. This is shown in Figure 9. A total of 54 segments out of 93 are included.

Table IV

T000 - SEGMENT USAGE (CASE I)

T_{3}	221	T_{41}	71	T_{85}	100
T_{5}	221	T_{42}	12	T_{87A}	88
T_{7}	209	T_{44}	233	T_{88}	47
T_{10}	85	T_{47}	56	T_{89}	17
T_{11}	440	T52	304	T_{90}	14
T_{13}	17	T_{53}	71	T_{91}	16
T_{14}	192	T_{54}	12	T_{93}	41
T_{19}	85	T_{57}	55		
T_{21}	233	T_{58B}	178		
T_{22}	209	T_{65}	304		
T_{23}	17	T_{67}	14		
T_{26}	146	T_{69}	57		
T_{27}	16	T_{72}	12		
T_{28}	42	T_{75}	41		
T_{29}	85	T_{76}	28		
T_{31}	83	T_{77}	2		
T_{32}	2	T_{78}	15		
T_{34}	233	T_{79}	100		
T_{37}	15	T_{80}	135		
T_{38}	2	T_{81}	41		
T_{40}	204	T_{84}	2		

(Remainder are not exercised)

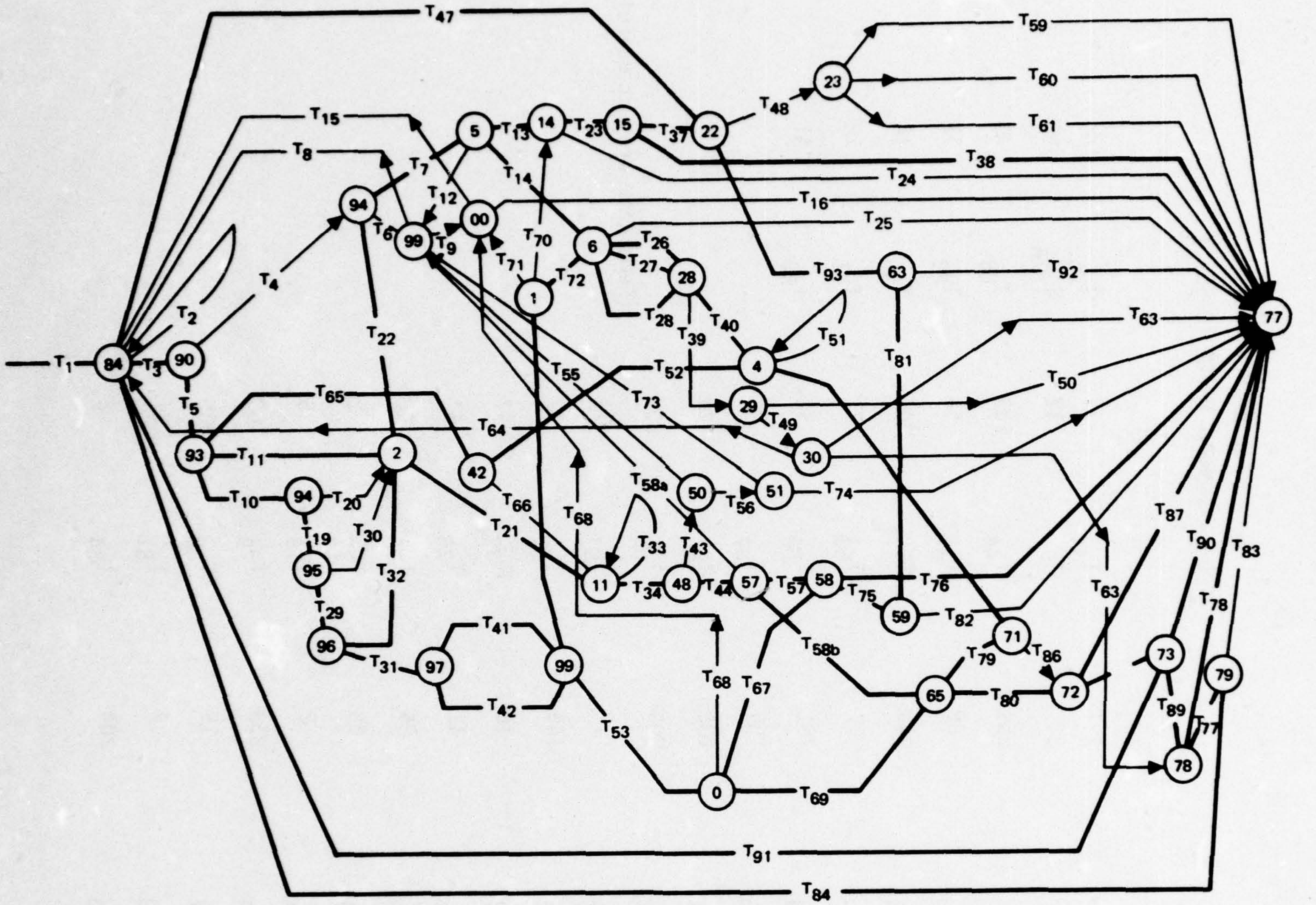

Figure 8. Paths Exercised by First Test Case from Uniform Distribution

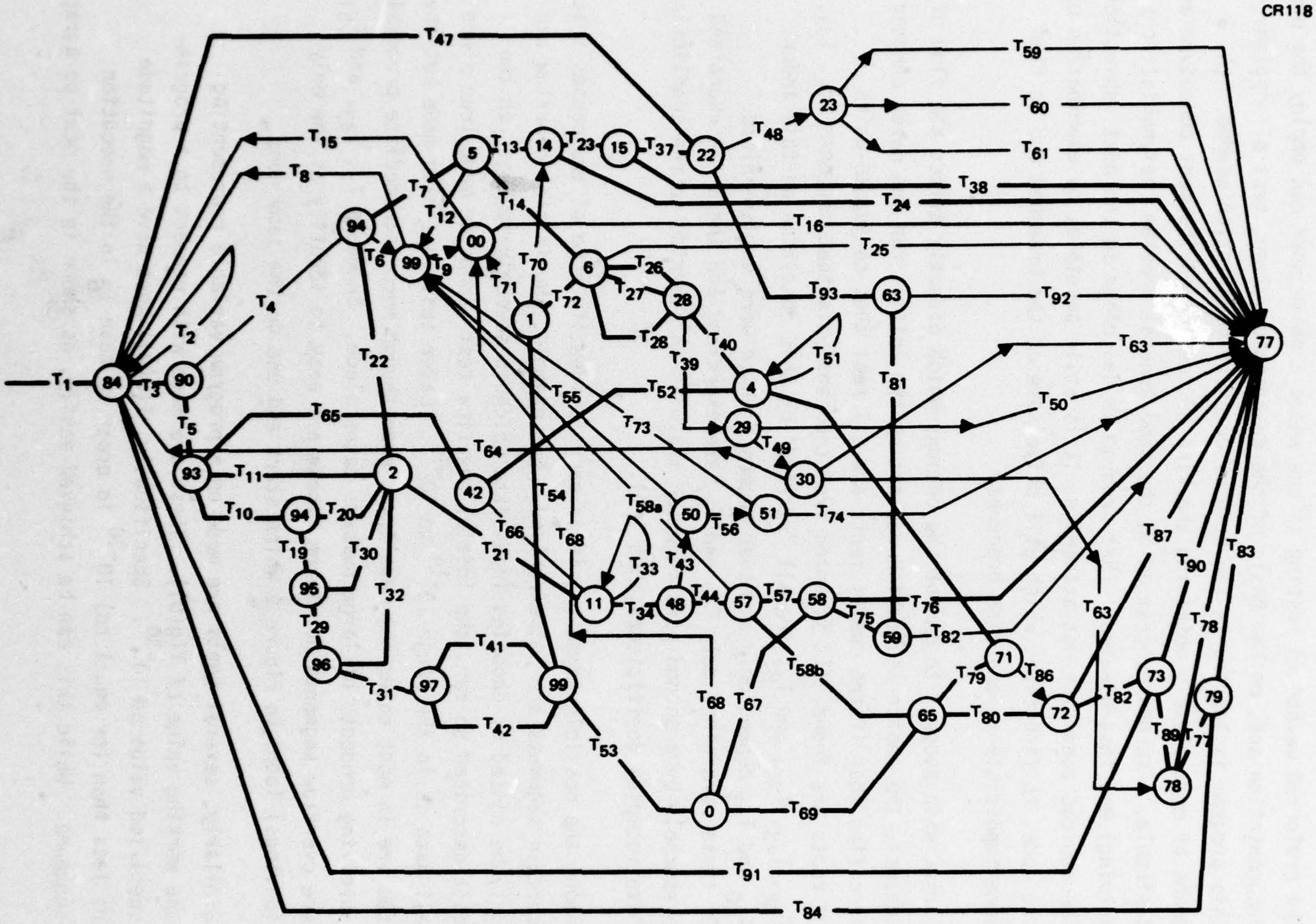

Figure 9. Ten-Sample Uniform Distribution Segment Usage

b. Special Test Cases

The preferred method of testing is one where a naive user can employ the program documentation and, on the basis of the description given, test all options. This program is lacking in the kind of detailed description needed. (As a means of more closely obtaining the kind of testing environment conceived as desirable, a second program using the same general problem (polynomial root solving) was instrumented. That program is described in the next subsection). The desired condition not attaining, it is still possible, by examination of the code, to find cases which will drive some of the branches and to find some impossible-to-exercise branches.

First, with respect to one of the options which directly affect the flow of control, the choice I=0 (which was set on the initial run and never changed) specifies that if the coefficients are all real then conjugate-pairs of roots are found. It is noted from code analysis that segments T_{15}, T_{24}, T_{50}, T_{73}, T_{76}, and T_{87} are all "started" by this condition on this index. T_{50} and T_{73} depend only on other segments which were not exercised. Of those remaining, only T_{76} and T_{87} are exercised and these are exercised presumably by a second pass in which there is an override on the conflicting data (complex coefficients, but I=0).

Among the possible segments, it is easy to identify some of the causes that certain segments are not exercised. For example, the initial overflow can only be caused by choosing input data which is severely taxing. In the data described so far, the initial overflow test cannot be made true since all data is in the range 10^{-14} to 10^{-14}. Later tests are also made but they too are in most cases not made true, even though much intermediate processing involving products of large numbers takes place. Segments T_2, T_{33}, and T_{51} are overflow segments not taken (these are easy to identify as the only (obvious) loops in Figure 9 which start and end on the same node.

Similarly, several tests are made on a program variable representing the working value of T(g(0)). These are made with respect to a program-specified value of 10^{-30}. Specifically, T(g(0)) must have a magnitude of less than (or equal to) 10^{-30} in order to have T_6 in the execution sequence. While this can be achieved easily, as shown in the next paragraph,

it would be (or would have been) convenient, to merely increase the value of tau (10^{-30}) to, say, 10^{-5} (the accuracy of the roots may suffer, but this is not a real consideration).

To continue with this point, it is noted by code inspection, that branch T_6 can be taken provided

$$|a_0|^2 - |a_n|^2 < 10^{-30}$$

a requirement met by setting $a_0=a_n$. This of course, "never" occurs for randomly generated data. So this case is one which essentially must employ examination of the code.

In an elaboration of this kind of case construction, examination of the code shows that the following list of segments which are not exercised but which are affected by "zero-relations" among the program variables at entry: T_{15}, T_{16}, T_{12}, T_{49}, T_{74}, T_{58A}, T_{68}, T_{71}. It is well to note here, however that not all of these are directly dependent on values of the input variables: some of them are transfer controls. Listed below are all of the segments which were (possible) and were unexercised after the 10-size uniform random drivers were used, with those of the above listed set, together with the overflow test branches mentioned above, either encircled (zero tests) or "ensquared" (overflow tests).

Table V

Unexercised Segments

$\boxed{T_2}$	T_4	T_6	T_8	T_9	T_{12}
T_{15}	T_{16}*	T_{25}	$\boxed{T_{33}}$	T_{39}	T_{43}
T_{48}	T_{49}*	T_{50}	$\boxed{T_{51}}$	T_{55}	T_{56}
T_{58A}	T_{59}	T_{60}	T_{61}	T_{62}	T_{63}
T_{64}	T_{66}	T_{68}	T_{70}	T_{71}	T_{73}
T_{74}*	T_{82}	T_{86}	T_{92}		

***Most of the zero-tests can be expected to be exercised since as noted they involve control transfers or the running down of the equivalent of index registers. Those are marked by an asterisk in the table. Others such as T_{12}, T_{68}, T_{58A} are dependent on continuous variables.**

(1) Fundamental Combinations

It is interesting to illustrate here the way in which efficient testing can be developed on the remaining segments. If the status represented by the Figures 8 and 9 above attains, and assuming T_6 has not been exercised, it is noted that the node numbered 99* (at the end of the arrow representing T_6) has the property that all arrows exiting from it (T_8 and T_9) are zero. This means necessarily, that all segments leading into that node are not exercised. Thus, for example, because T_8 and T_9 are not exercised it can be ensured that T_{12}, T_{73}, T_{55}, T_{58A} are not exercised.

This forms the seed of a method for searching for "fundamental" combinations of segments, and although not explored in the present contract seems to be of promise in an interactive mode of test case selection.

These combinations can be found from the list as constructed in Table V, by locating segments with the same start node which all have zero usage.

The following combinations are found to be linked in the manner suggested

$T_8 - T_9$	:	T_6, T_{12}, T_{73}, T_{55}, T_{58A}
$T_{15}-T_{16}$	:	T_9, T_{68}, T_{71}
$T_{49}-T_{50}$	:	T_{39}
$T_{55}-T_{56}$	:	T_{43}
$T_{59}-T_{60}-T_{61}$	:	T_{48}
$T_{62}-T_{63}-T_{64}$	:	T_{49}
$T_{73}-T_{74}$	:	T_{56}

*Because of (self-imposed) display constraints, there are a few nodes with non-unique numbers - there are two 99-labeled nodes, for example.

This formation would be very easy to program and would be very useful even though, in some respects, it is backward, for it does not always specify which of multiple drivers will produce the necessary data to drive the segments. On the other hand, as the last five combinations show, the drivers may be clear but the "driven" is not. (It is fundamental that the out-degree of every node of the graph-representation of the program by segments is 2 or more, and that this ambiguity is therefore inevitable).

In developing a strategy for testing, the nesting of the combinations can produce the likely best choices. Thus, T_{43} if exercised will exercise either T_{55} or T_{56}, and, in turn, either T_8 or T_9 in the former case, or T_{73} or T_{74} in the latter case. T_{39} will exercise either T_{49} or T_{50}, and the former would exercise T_{62} or T_{63} or T_{64}. In developing a metric for the degree of testing achieved, it is well to realize that T_{43} "represents" 3 segments which will be exercised, similarly T_{39} "represents" 3.

(2) Special Cases

The only segments which are not among the set of fundamental combinations and are not overflow or zero-dependent are: T_{25}, T_{66}, T_{70}, T_{82}, T_{86}, T_{92}. These are discussed as "special cases" here, but of course, the fundamental combinations and zero- and overflow-dependents are also special cases.

T_{25}: Analysis of this shows that this segment requires T1 (a "working" value of T) to be greater than zero, <u>OR</u> CNORM>0, <u>AND</u> the index variable KL=1,2, or 3. The first two conditions are (probably) met. Entry To T_{25} comes from either T_{72}, which, if exercised at all, "carries" one of the values K=4,5, or 6; or from T_{14}, which does not change KL, and which comes from T_7 which again does not change KL, and which, in turn, comes from T_4 or T_{22}. But T_4 is overflow dependent, as noted earlier, while T_{22}, which was exercised, can only "carry" the value KL=4.

This means that T_{25} can only be exercised on an overflow, along with T_4 and T_2. (Note again that T_2, T_4 and T_{25} are (probably)linked; however, in a less certain way than is the case with the fundamental sets).

T_{66}: This segment requires the value MY=2 AND T_{52} to be exercised. But MY=2 is set only by one instruction (408) which is contained only in T_{51}, which is an overflow path. Thus, T_{66} is linked to the overflow segment, precisely as with the preceding case.

T_{70}: The segment requires KL=4, 5, or 6. Thus, T_{41}, which requires KL=1,2, or 3 cannot be the driver and T_{42} is the only potential source. It does carry KL=4,5 or 6. The code shows that failing a test for magnitude on the real and imaginary parts of a_0 and a_n, these are all multiplied by 10^{10} and a new test is made on the difference. It is reasonable to expect that this segment would have been exercised.

The code on instruction numbered 398, indeed, does look wrong: the code which reads CIMAG**2, should be CIMAG(1)**2. (Whether or not this is a real or only an apparent error was not being investigated; it is noted that the original code is the same and it is not a typing-translation error).

T_{82}: This segment depends on some rare but not impossible computational values. It is necessary for KL=3 and this is attained. Additional cases should exercise this branch.

T_{86}: This segment depends on the predicate L>N being true. (For the case at hand, N=4). On the other hand, T_{86} depends exclusively on T_{79}. But in order for T_{79} to be exercised, it is necessary for L to be less than N. But L is increased by 1 unit at a time; hence, the case L=N is reached ahead of the test L>N and T_{86} can never be exercised.

T_{92}: This branch requires a loop counter in excess of 20. For the data used, this did not happen. Each time the subroutine is entered the counter is set to zero in initialization and it is set to zero on exit. This branch should be exercised in a different sample.

(3) Constructed Cases

1. The first case consisted of assigning as input a 1st degree polynomial ($-.004z+3=0$) with real coefficients, without error check. This simple choice exercised only 24 segments of T000; but among these are the segments T_{39}, T_{49}, T_{50}, T_{62}, T_{63}, T_{64}, none of which had been exercised by the random tests. The string T_{49}, T_{62}, T_{63}, T_{64} form a fundamental combination and T_{39}, T_{49}, T_{50} form another.

2. This case was a 1st degree polynomial with complex coefficients, specifically $(-.004+3.2i)z+(3.0-.07i)=0$. No error check was used. This caused a subset of the case 1 segments to be exercised.

3. A second degree polynomial specifically $10.8z^2-.004z+3.0=0$, with real coefficients, roots computed without conjugates, and no error checks, resulted in one new segment, T_{92}, (as well as T_{83} which had been exercised by random numbers).

4. This consisted of a polynomial of degree 2, with complex coefficients without error check and without conjugates. No new segments were exercised.

5. This case consisted of a degree 2 polynomial. The "conjugate" choice was made: that is, one root is found by taking the conjugate of the other. An error bound of $r=10^{-14}$ was employed. No new segments were executed.

6. A degree 3 real polynomial, no conjugates, no error check comprised case 6. Segments T_{29}, T_{79}, and T_{85} were exercised, but these had been exercised by at least one of the random cases. (In addition, for summary comparison purposes, 6 new PTT-segments in the LEHMER and S000 were exercised).

7. A complex polynomial of degree 3, without error checks and without conjugates, was used. Segments T_{82}, T_{84}, and T_{21} were new segments. T_{84} and T_{21} had been exercised by random numbers. (Seven PTT-segments among the LEHMER or S000 were exercised).

Case 8 - A 4th degree real polynomial without conjugates and no error check was used. T_{32}, T_{91} were exercised, both of which had been exercised by the random cases (Three PTT-segments).

Case 9 - A 4th degree complex polynomial without error check, and without conjugate was used. No new segments were exercised.

c. Status of Testing (Random and Selective)

T_2: Overflow is required

T_4: Overflow dependent

T_6: Exercised with $a_n = a_o$

T_8: Exercised when conditions on T_6 are met.

T_9: Can be exercised by a polynomial with roots which have a small magnitude, but whose coefficients are sufficiently large to exceed the value of tau (10^{-30}).

T_{12}: Requires $T_1 = 0$, but, if this were to occur, the segment T_6 (starting at 294) (and not T_7) would be exercised. But T_6 and T_{12} are not compatible. T_{12} CANNOT BE EXERCISED.

T_{15}- Exactly one of these will be exercised when T_0 is exercised.
T_{16}: The former would be exercised when conjugates are specified and the latter when they are not.

T_{25}: This was discussed above under the section title, Special Cases. This is overflow dependent (on T_4).

T_{33}: This segment is an overflow test branch.

T_{39}: This was exercised by the case of a first degree polynomial (Constructed Case No. 1).

T_{43}: The conditions are the same as for T_9.

T_{48}: This would be exercised when conjugates are specified and would be exercised by Constructed Case No. 1, if that option were specified.

T_{49}: This is exercised by Constructed Case No. 1.

T_{50}: This is exercised by Constructed Case No. 1.

T_{51}: This is an overflow test branch.

T_{55}: This segment would be exercised with T_9 on the first pass through a loop: T_{55}-T_9-T_{15}-T_3-T_5-T_{11}-T_{21}-T_{34}-T_{43}-T_{55}.

T_{56}: This segment would be exercised on the fifth passage through the same loop. A higher degree polynomial would exercise this segment.

T_{58}: This requires a working value of T(g(0)) to be zero. This can only be achieved by an extremely rare event. While not impossible to exercise, it is very unlikely that it will be.

T_{59}: This segment requires T_{48} to be Exercised and KL=1, 2, or 3. T_{48} is conjugate choice dependent. This segment will be exercised with a higher degree polynomial.

T_{60}: This requires KL=4, and since it can be achieved with almost any choice on initial input. T_{48} must be driven, however.

T_{61}: This requires KL=5 or 6 which can be met when T_{48} is driven.

T_{62}: Exercised by Constructed Case 1.

T_{63}: Exercised by Constructed Case 1.

T_{64}: Exercised by Constructed Case 1.

T_{66}: This segment is described above. It is linked to overflow.

T_{68}: This segment requires CNORM=0 and KL=1, 2, or 3. With well chosen coefficients this will be exercised from the top.

T_{70}: This segment has been discussed above under Special Cases. There may be a coding error in the program.

T_{71}: This segment requires CNORM=0 and KL=4, 5, 6.

T_{73}: This requires I=1, and will be exercised with any data.

T_{74}: This requires I=0 and would be exercised when T_{56} is

T_{82}: This was exercised by the 7th Constructed Case.

T_{86}: As shown in the analysis under Special Cases, this segment CANNOT BE EXERCISED.

T_{92}: This segment, discussed above under Special Cases, willbe exercised with a higher degree polynomial.

d. Summary of Analysis

First, with respect to the quite limited testing which was done, it is noted that, excepting the overflow or overflow-dependent segments, those which are dependent on the choice I=1 and which would have been exercised with that choice, those which cannot be exercised, and those which are directly dependent on these, the number of segments exercised by random testing with the small sample of 10 exercised 54 of 80 possible segments.

It is useful in this respect to note that the usual problem-proofing procedure consists of running a set of simple cases (check problems) such as those listed under Constructed Cases. It is noted that the union of segments exercised by one or more of these tests numbered only 16. The total possible can again be taken to be 80.

From the results of this simple comparison it seems clear that the random or blind testing (although to be sure other Constructed Cases - especially higher degree polynomials - would produce a higher yield than those employed), is very effective in testing a program.

Presented in Figure 10 is a modified flow diagram in which three essential changes are made to the original diagram: the impossible segments are deleted, the conjugate-choice option corresponding to I=1 are eliminated, all overflow or overflow dependent segments are deleted. In addition, an exit on the occurrence of a zero or negative degree is deleted along with the segments which (after the previously described deletions) are dependent only on that condition (segments T_{39}, T_{49}, T_{62}, T_{63}, and T_{64} are so affected).

This figure shows clearly the degree to which these random tests exhaust the possible branches. It is also clear from the figure what additional tests need to be developed. On application of Constructed Cases which are discussed above, the segment T_{92} and T_{82} were exercised. It is again clear where the focus should be placed for the additional segments. For example, T_{43} if exercised, will add a minimum of 2 and as many as 3 segments.

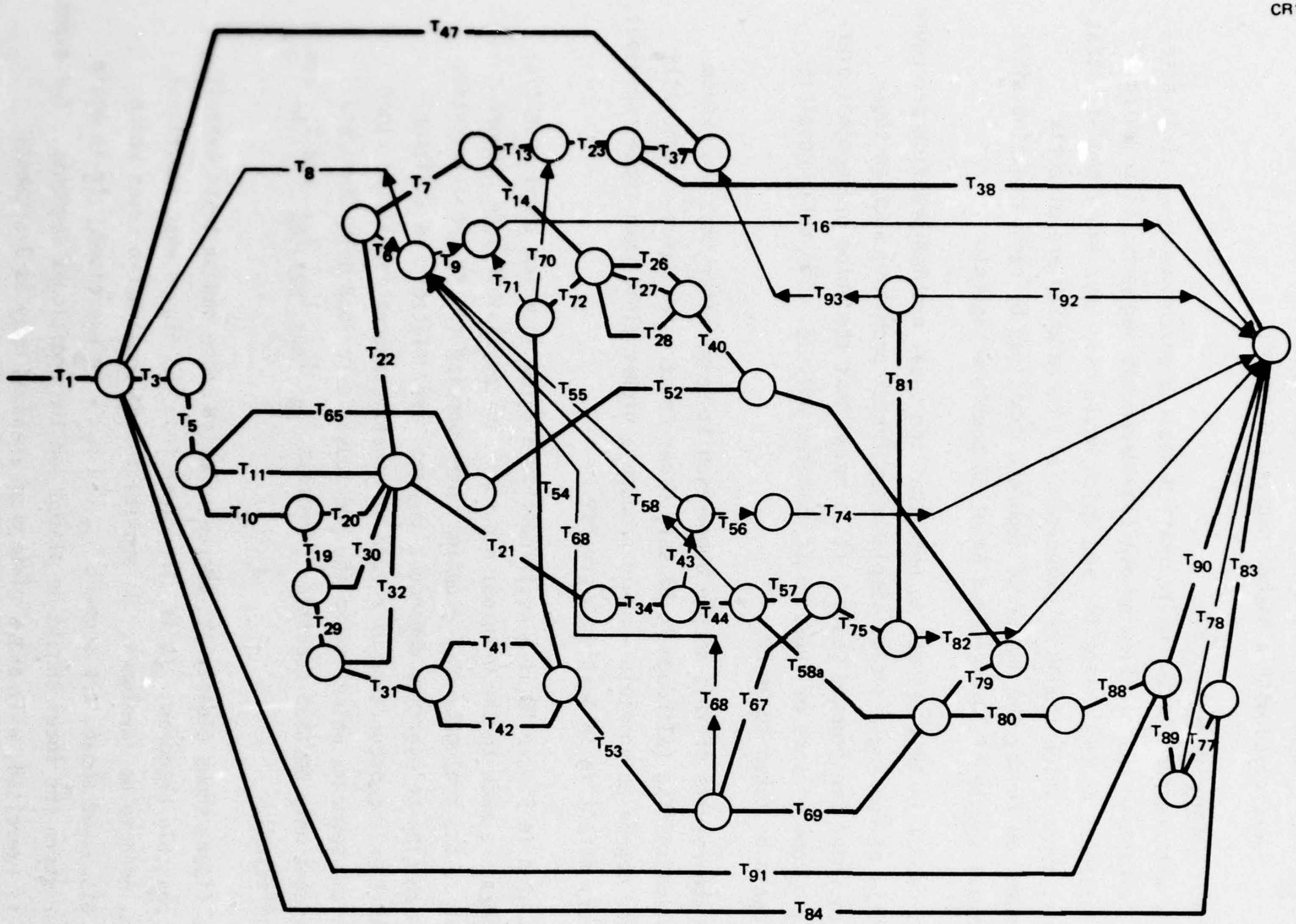

Figure 10. "Pruned" Flow Diagram and Usage

B. IMSL Routine ZPOLYR

1. Description of Root-Solving Method

Because of the difficulty in tracing through the unstructured program which was written for the Lehmer method, a second polynomial solver was selected using the criterion that the documentation be more adequate and the listing more easy to follow. This was supplied by an IMSL Library program named ZPOLYR.*

This program uses the Jenkins-Traub (16) three-stage algorithm. Some of the difficulties which the original program had are discussed in the analysis to form constructed cases. The major deficiencies seem to be: in the original program the largest roots were extracted first and the resultant "deflated" polynomial - obtained by factoring out a quadratic - on occasion, produced apparent zero-valued roots; the degree limitation was raised to 100 in the new or replacement program; zero leading coefficients cause an instant error exit.

2. ZPOLYR Program Listing

This program is commercially available and the listing is available in the IMSL Library.

3. Analysis of PTT-Segments

a. Summary Data

In the first run, a sample of size 10 from a triangular distribution was used. The range of this distribution on the logarithm of the coefficients was [-14, +14] with a mode of +2. Signs were chosen by parity of random numbers. The degree was chosen to be 4. The percentage of segments exercised by these cases (PTT-segments were used) were as shown in Table VI.

*This program and a major subroutine were both replaced by IMSL in 1975.

Table VI

ZPOLYR Segment Usage (Run #1)

Case No.	% Used	Number Used
1	44.68	63
2	44.68	63
3	65.96	93
4	65.96	93
5	50.35	71
6	44.68	63
7	54.61	77
8	65.25	92
9	65.96	93
10	44.68	63

Run #2 used the same degree (4), and a uniform distribution on the logarithm of the coefficient over the range [-14, 14]. A sample of size 20 was used.

Insofar as segment usage is concerned, the results were as shown in Table VII.

Table VII

ZPOLYR Segment Usage (Run #2)

Case No.	%	No.	Case No.	%	No.
1	60.28	85	11	39.72	56
2	62.41	88	12	54.61	77
3	61.70	87	13	65.25	92
4	65.25	92	14	63.83	90
5	54.61	77	15	46.81	66
6	46.10	65	16	52.48	74
7	45.39	64	17	39.72	56
8	46.81	66	18	69.50	98
9	50.35	71	19	68.09	96
10	60.28	85	20	54.61	77

The numbers here shown are both larger (69.50) and smaller (39.72) than for the 10-sample triangular case. This effect is probably more due to the distributional difference, than to the sample size. The uniform distribution can pick large values (1 to 10^{14}) for the coefficients as frequently as small values (numbers in the range 10^{-14} to 1).

Examination of the printout for the two cases (11 and 17) which exercised the fewest number, show a lack of convergence: "La Guerre's Method has failed to converge." The input data in case 11 showed a coefficient of about 1.5 x 10^9 for the z^3 term and 1.0 x 10^{-6} for the z^4 term and -4.5 x 10^{-4} for the coefficient of z. Case 17 had coefficients: 6.5 x 10^{-14}, 1.2 x 10^3, -4.9 x 10^{-8}, 1.1 x 10^2, 2.4 x 10^{-12}. The apparent conclusion is that because of a failure to converge, there was a quick exit.

Actual segment usage data for the Case 11 shows (a typically) zero usage in the program from instruction 132 to 195 (assuming continguity of numbers implies something about the actual operating sequence), corresponding to a program section which "extracts" a quadratic factor from the original or reduced polynomial. Entry into this section is governed by the test as to whether or not the nth iterative approximation is "close to the real axis relative to step size." Case 17 shows a similar usage pattern.

Before going into an analysis of the program and construction of special cases, the several additional run results are presented.

Run #3 employed a quadratic with 15 samples and a uniform distribution. All samples exercised the same (small) number of segments, 12. Furthermore, they all exercised exactly the same segments. Part of the reason for the light usage is mentioned above; basically it is the degree of the polynomial which causes the light usage.

Run #4 was characterized by choosing a polynomial of degree 12, uniform distribution on the log of the coefficients over the same range employed before, and a sample size of 15. Table VIII summarizes the usage data.

Table VIII

ZPOLYR Segment Usage (Run #4)

Case No.	% Used	No. Used	Case No.	% Used	No. Used
1	65.25	92			
2	68.09	96	9	67.38	95
3	69.50	98	10	40.43	57
4	66.67	94	11	69.50	98
5	62.41	88	12	68.79	97
6	67.38	95	13	68.09	96
7	65.25	92	14	68.09	96
8	64.54	91	15	64.54	91

Run #5 employed 40 samples, a 4th degree polynomial, and a uniform distribution as before. Table IX summarizes the usage data for the run.

Table IX

ZPOLYR Segment Usage (Run #5)

Case No.	% Used	No. Used	Case No.	% Used	No. Used
1	39.72	56	21	43.97	62
2	45.39	64	22	46.81	66
3	44.68	63	23	68.09	96
4	45.39	64	24	36.17	51
5	61.70	87	25	44.68	63
6	46.81	66	26	49.65	70
7	69.50	98	27	60.28	85
8	46.81	66	28	45.39	64
9	47.52	67	29	61.70	87
10	44.68	63	30	44.68	63
11	44.68	63	31	44.68	63
12	46.81	66	32	46.81	66
13	45.39	64	33	63.83	90
14	50.35	71	34	70.92	100

Table IX
(continued)

Case No.	% Used	No. Used	Case No.	% Used	No. Used
15	56.03	79	35	44.68	63
16	44.68	63	36	60.28	85
17	69.50	98	37	62.41	88
18	44.68	63	38	62.41	88
19	60.28	85	39	69.50	98
20	63.12	89	40	51.06	72

Finally, a run with a 15th degree polynomial and a sample size of 15 produced the summary usage data depicted in Table X.

Table X

ZPOLYR Segment Usage (Run # 6)

Case No.	% Used	No. Used	Case No.	% Used	No. Used
1	68.79	97	9	67.38	95
2	64.54	91	10	68.79	97
3	61.70	87	11	67.38	95
4	65.25	92	12	65.25	92
5	65.96	93	13	65.96	93
6	65.25	92	14	64.54	94
7	65.96	93	15	67.38	91
8	68.79	97			

b. Estimation of Total Number of Execution Sequences

(1) Introduction

The interesting pattern in the above data is the separation between the trial numbers at which the "new" or apparently "new", execution sequences are found. If, as is initially assumed each execution sequence (or realizable logical path)

has the same chance of being exercised when random numbers are used as input, then the number of these paths can be estimated by applying a model developed by Jelinski and Moranda for a different purpose. That model was described briefly in Section III.There are some necessary changes in interpretation, however: in the original model, the number of software errors contained in a package is the equivalent of the number of execution sequences here. Time, which was the independent variable in the original model, is made to be the analog of trial number in the variation. When an error is detected and corrected, the original model assumes a detection rate which is correspondingly reduced (by one unit); in this application a new execution sequence is noted by comparing the pattern of the segments exercised against all earlier occurring sequences.

Because execution sequences are characterized by the particular segments driven (and the order in which they are executed) the total number listed in the preceding tables are not sufficient. They are, of course, very useful since if the totals are different then necessarily the execution sequences are different.

There is a pair of fine points which need to be acknowledged and which indeed help in the definition of what is here called an execution sequence. First, it is possible for the PTT-segment usages to be identical in two cases while the actual sequences could differ; no information is available to decide such fine differences. Second, two cases may exercise exactly the same PTT-segments but with different numbers on one or more of the segments and these are classified as the same. For many purposes, including the purpose of thorough test, both of the interpretations are acceptable variants.

(2) Technique

For the analysis given here for illustration of a technique, the runs 2 and 5 are merged together to form a sample of size 60. These are numbered from 1 to 20 through the Run 2 sample and 21-60 through Run 5. As noted above, it is easy to determine by inspection of summary data, the first nine cases represent different execution sequences. Case 10 not only has the same % usages as Case 1, but identical segment usage. Case 12, on the other hand, has a different execution sequence from Case 5, even though they

have the same total segment usage. Similarly, Case 13 is different from Case 4 although they have the same total count. Taking each case number in sequence, if it is different in total from all the preceding, it represents a new execution sequence; if it is the same as one or more earlier-listed ones, it must be closely examined against all of them with the same usage %'s, to see if it indeed, is a different case.

The following sequence of numbers is formed as follows: a 1 is recorded if there is a new sequence found and a zero if there is not. The sequence of 0's and 1's versus case number is:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	0	1	1	1	1	0
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0	1	1	0	0	0	1	0	0	0	1	1	1	0
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
0	1	1	1	1	0	1	0	0	1	1	1	0	1	0
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	0	0	0	0	0	0	0	1	0	0	0	0	0	1

The formulas developed for the De-Eutrophication model are:

$$\sum_{i=1}^{n} \frac{1}{N-(i-1)} = \frac{n}{N - \frac{1}{T} \sum_{i=1}^{n} (i-1)x_i} \tag{11}$$

$$\phi = \frac{n}{NT - \sum (i-1)x_i} \tag{12}$$

In the present application, n will stand for the number of execution sequences found during a number of cases T (one case per unit of time). N is the unknown number of execution sequences (not to be confused with the polynomial degree which has the same notation): the number which could eventually be exercised by a 4th degree polynomial whose coefficients are chosen at random in the manner indicated. X_i represents the "time" between the discovery of new execution sequences. If this happens in consecutive cases, this number

is taken to be 1. Ø is a proportionality constant and represents "one execution sequence" worth of detection rate (starting with a value NØ for the detection rate and decreasing by one on each discovery.)

The data so defined becomes

$X_1=1$, $X_2=1$, $X_3=1$, $X_4=1$, $X_5=1$, $X_6=1$, $X_7=1$, $X_8=1$, $X_9=2$, $X_{10}=1$,

$X_{11}=1$, $X_{12}=1$, $X_{13}=2$, $X_{14}=2$, $X_{15}=1$, $X_{16}=4$, $X_{17}=4$, $X_{18}=1$,

$X_{19}=1$, $X_{20}=3$, $X_{21}=1$, $X_{22}=1$, $X_{23}=1$, $X_{24}=2$, $X_{25}=3$, $X_{26}=1$,

$X_{27}=1$, $X_{28}=2$, $X_{29}=2$, $X_{30}=8$, $X_{31}=6$.

If it were assumed that only the first 20 cases (i.e., the Run #1 data) were available, then the following processed data would apply: X_1-X_{15} as above, $X_{16}=2$ (at least 2 units to the next detection, based on the best "current" knowledge - that at trial number 20); n=16; T=20. This results in a ratio $\Sigma(i-1)X_i/20 = 8.4$ and this ratio completely determines the parameters N and Ø.

Tables have been constructed to solve those equations and are found in Appendix II. From these tables the following information is found for the ratio (8.4) and n=16.

$$\hat{N} = 31.82$$
$$\text{Var}(\hat{N}) = 29.54 \text{ (standard deviation of 5.4)}$$

The total number found after 60 cases is 31, an agreement so close as to require explanation. It is noted that "time" has by no means run out and the 60th case showed a new discovery. Hence, the total error content is larger than 31, and the agreement is fortuitous. By way of further illustration and to erase the impression that this technique is as accurate as luck has made it appear at first sight, the first 40 trials are analyzed. Again, the data at X_1-X_{15} is as before, X_{16} is now "revealed" to be 4 (instead of the value 2 which served as the current best value before), the values X_{17}-X_{25} are as listed.

Computation of the ratio $\sum_{i=1}^{25} (i-1)X_i/40$ is found to be 13.4. From the tables of Appendix II, the estimate for N in this case, is 50.25 and the

standard deviation of the estimate is 6.1. The realized (so far) value of 31 falls barely short of the 3σ lower limit of acceptability, but of course would increase with larger samples. Computation with the total sample of 60 produces for n=31, a ratio of 18.46 and an estimate of 40.5 with a standard deviation of 2.95. This is the best estimate and is certainly a reasonable one. In the long run, and 60 is not considered long, this is the expected number which will be exercised in this way. The value obtained is noteworthy in at least two respects. The conventional myth about execution sequences is that there are a very great number (10^{10} for even reasonable sized programs) of them. The evidence seems to indicate that for this relatively small program this is not the case. On the other hand with 141 segments with which to form sequences and a rough estimate of 70 two-way predicates to consider, there are reasons to believe that there would be a great number of sequences. Again, this does not seem to be the case. Of course, this observation must be put in context. The choice of the degree as 4 in all of the above analyses may have prevented a great number of the 141 segments from being used. (This effect is very clear for the quadratic - Run #3 showed all 15 cases had exactly the same segment usage).

This point was investigated with inconclusive results. An analysis of the segment usage data for a run made with a polynomial of degree 12 (Run #4), produced the following pattern of 1's and 0's (using the above interpretation):

1,1,1,1,1,1,1,1,1,1,1,1,0,1

indicating, at first look, a large number of sequences. But the initial segment (of 14) of the corresponding sequence in the preceding analysis showed a similar pattern but produced many duplicates in the next 10 cases. If the data from Run #6 (not quite comparable because it inputs a polynomial of degree 15) is used together with the Run #4 data, there is seen to be indications of duplicates (within the run (#6), and with that of Run #4). Closer analysis was not made.

It seems fairly sure that the number of execution sequences is much smaller than is ordinarily thought.

(3) Summary and Extensions

The technique employed above in the illustration provides the only known practical means of estimating the number of execution sequences. The alternative procedures involve the solution of a number of simultaneous logical equations formed from the program's predicates, and result in a very large number of equations and no clear solution in case the program functions (those that change program variables) are non-linear.

This technique has two variants which amount to application of models developed by Moranda [13] and [14]. The first model treats detection of errors (or new execution sequences in the present case) as random a process in which the detection rate decreases in a geometric progression on the occurrence of each error detection (and, correction in the case of errors).

Because of the importance of this and the earlier-described model, they were more fully developed, particularly in the direction of obtaining variances and covariances of the estimates and, preparing tables for the first named model, which will greatly assist the task of solving a difficult equation. The second model results in equations which are not solvable in advance, since as seen in Section V, they are based on polynomials with random numbers as coefficients.

The third model also developed by Moranda [14] has applicability to the process of estimation of the number of execution sequences. This model, described in Section V, while decreasing the rate in a geometric progression has a constant rate which represents the long term or asymptotic detection rate. That model while having applicability in describing the transition between the "burn-in" and steady state phases as defined in hardware reliability studies, has applicability to software which is used in programs controlling equipment, and which consequently derive some of their input from sensors of the ambient conditions.

In more extensive use of the above described techniques, it is well to set up separate tests for each combination of input parameters. For example, the degree of the polynomial is known to have an pronounced effect on the execution sequences which are realized. That has been noted in

the above (all quadratics exercised but 12 segments). For the program which is tested here, a good analysis would involve estimating the execution sequences for each degree, from zero (constant), to at least 80 . (Instruction 4-5 makes a test on the degree against an upper limit of 79).

4. Constructed Cases (ZPOLYR)

The following PTT-segments were not exercised by any of the preceding described runs:

2	4	14	16	21
21	24	25	27	28
29	30	39	41	42
44	45	48	53	92
94	109	111	130	132

Because of the extensive comments in the listing of the program, it is quite easy to construct cases which exercise many of these still undriven segments.

a. <u>Segment Analysis</u>

For a polynomial of degree 80, PTT-segment No. 2 (denoted Z_2, here) is exercised, while it is not for any smaller degree. This or any larger degree causes the execution sequence Z_2, Z_4, Z_{140}. Of these, Z_{140} is exercised by any error message and it has been exercised before. Thus, the limitation on the degree causes two new segments to be exercised.)

A polynomial of 1st or 0th degree will cause the execution sequence Z_1, Z_3, Z_5, Z_6, Z_9, Z_{35}, Z_{37}, Z_{40}, Z_{139}, all of which are exercised in at least one prior case (if the quadratic had been considered after this construction, Z_9 and others would not have been exercised.

Segment Z_{14} can be exercised by entering a string of zeros for input. This segment then leads into formerly exercised segments.

Segment Z_{16} can be exercised by entering coefficients which exceed a programmer-specified parameter, which in the cases employed had the value 10^{150}. This did not and could not occur with the "picking" function used. It can be by choosing $2 \leq N \leq 12$ and any coefficient in excess of 10^{150}. Alternatively, the parameter can be assigned a smaller value.

Segments Z_{21}, Z_{23}, and Z_{24} are inexorably linked and can be exercised by back-working from the condition ABS (SNGL (DU(I)) < BIT), where BIT is set to be the smallest positive number in the machine. This condition can be met by setting the leading coefficients of the polynomial equal to zero. The implication to the program user is that an infinite zero is found (an error message would have done as well for ordinary problems, but since ZPOLYR is a callable subroutine it is useful to have this capability).

Segments Z_{25}, Z_{27}, Z_{28}, Z_{29} and Z_{30} are linked and are dependent on a condition met by the occurrence of zero for at least two leading coefficients (i.e., the coefficients of z^n and z^{n-1} are zero). This will occur with very low probability (the product of two very small values) when random numbers are used.

Segments Z_{39}, Z_{40}, and Z_{41} are linked and dependent on the occurrence of an error or anomaly in a called subroutine (ZQUADR). Insofar as this routine is concerned, these segments should not be counted as relevant.

Segment Z_{42}, Z_{44}, and Z_{45} are linked and depend on the constant term of the input (or derived polynomial) being zero. With random number generators used to pick coefficients, this is unlikely and indeed it did not occur. Entry of a 4th degree polynomial with a zero for the constant would exercise Z_{42}, Z_{44} and Z_{45}.

Segment Z_{48} requires simultaneous (logical AND) satisfaction of a predicate which states that the original polynomial, or a "working" polynomial derived from it by quadratic factoring, has the (N-2)nd and (N-1)st coefficients equal to zero. This can be met most easily by choice (and not "easily" by random selection).

Segment Z_{53} requires three (logical AND) conditions to be satisfied:

(a) The degree of the polynomial must be even.

(b) The product of the signs of the coefficients of the lowest and highest degree terms in a reduced polynomial must be positive.

(c) The occurrence of zero for the leading coefficients of a polynomial when returning from a subroutine (ZQUADR).

This segment had no realized predecessor for all but one case; that is to say, the lead-in path to the segment was blocked except for one case. In that one case there was an overflow in the program and the real part of the first root extracted had a magnitude of 10^{151}. This probably caused the deflated polynomial, obtained by extracting a linear or quadratic factor out of the original polynomial, to appear to have zero leading coefficient and the test conditions (c) was met. However, the product of signs in the original polynomial (and probably in the deflated polynomial), in that case was negative. This path can be exercised by random numbers but was not.

Z_{92} and Z_{94} are linked and depend on an error occurring in the subroutine ZQUADR. This did not occur at that point in the program. These segments should not be considered relevant to the ZPOLYR subroutine.

Z_{109} and Z_{111} are exactly the same as the preceding pair: they depend on an error in ZQUADR. These too should not be considered as relevant.

Z_{130} and Z_{132} are linked and depend on a zero value for the imaginary part of a coefficient of a working polynomial. This is not likely to occur in random tests.

Z_{138} depends on an error occurring in ZQUADR and is not relevant.

b. Summary

From the above, a categorization can be made. First, those that test for acceptable data:

Z_2, Z_4, Z_{16}

Second, there are those that are zero test dependent:

$Z_{14}, Z_{25}, Z_{27}, Z_{28}, Z_{29}, Z_{30}, Z_{42}, Z_{44}, Z_{45},$
Z_{48}, Z_{130}, Z_{132}

Third, there are segments which are irrelevant in that they depend on the occurrence of an error in a called subroutine:

$Z_3, Z_{39}, Z_{41}, Z_{92}, Z_{94}, Z_{109}, Z_{111}, Z_{138}$

Finally, Z_{53}, stands apart and requires satisfaction of three conditions. It can conceivably be exercised by random data.

The number of exercisable segments by random data then can be considered to number 131.

5. Rationale for Test Case Selection (ZPOLYR)

The difficulties which occur in the preceding described program can or could be alleviated in several ways.

First, as a general comment, it is clear that a systematic case selection which consists in stepping through the degree of the input polynomial would accomplish the testing of those branches which are degree dependent.

The same or similar remark would apply to other input parameters, which are actually programmer options, but which in most cases are not generally changed from the built-in values which are provided when no choice is made by the user. Varying these would provide a more exhaustive level of testing even though the computational results may be inaccurate, or intermediate results may cause premature or unnecessary overflows.

For a fixed degree and a fixed set of input parameters (as distinct from input data) the results which have been obtained are more than satisfactory: random cases in small sample sizes generally exercise all but the overflow and zero-tests for program variables. This cannot be compared with other options until a standard set of input cases is specified. But it is clear that the cases used comprise a more extensive testing level than "ordinarily" is achieved. Ordinarily the test cases are made by forming simple integer-coefficient polynomial and generally with ratios of max to min which are small while in the random testing these ratios can (and did) achieve values of nearly 10^{28}.

In order to supply a more exhaustive test and to accomplish a hybrid random/constructed case selection, a sequence of polynomials can be formed in which the value zero is provided as a part of the domain from which the random numbers are chosen. As noted before, the random testing employed in the illustrated cases could not pick zero values, and even if zero were in the range of

possible values it would be selected with essentially zero probability. This fact, coupled with the realization that the particular nature of the distribution has no real relevance to exhaustive testing, make this option very appealing. The probability split between non-zero and zero input cases is (again) arbitrary but if the split is too much biased against zeros, the occurrence of multiple zeros such as are required for certain program branches, would not occur often in reasonable sized samples. A split of 8 to 1 seems reasonable for input samples of reasonable size.

C. Curvature Program

Because of the large non-stochastic portion of the testing in the polynomial-root-solver programs which were discussed above, a library search for programs which were less-dependent on program parameters was made. The following described program is seemingly ideal in this respect.

1. Description of Program CURVTR

The program selected computes the curvature, and the direction cosines of the normal, at a point or a surface defined over the unit square (using variables u and w as the independent variables, this is the region $0 \le u \le 1$ and $0 \le w \le 1$).

Each coordinate of the surface is represented by a bi-cubic in the variables u and w. Thus the x-component is

$$x(uv,) = \sum_{i=o}^{3} \sum_{j=o}^{3} A_{ij}u^i w^i$$

and similarly for the y and z coordinates of a surface point. The program employs the notation v, for the point with components x,y, and z. There are altogether 48 coefficients for the three coordinates. The so-defined surface is unique and unambiguous.

Required for the curvature and direction cosines are the first and second partial derivatives with respect to the parameters u and v and these are determined by simple formulas. For example,

$$\frac{\delta x(u,v)}{\delta u} = \sum_{j=o}^{3} \sum_{j=o}^{3} iA_{ij}u^{i-1}w^j.$$

Essentially all of the computations are formula evaluations with a pair of assigned values to u and w.

2. CURVTR Listing

The listing of this program is shown in the following pages, as Figure 11.

3. Random Testing of CURVTR

Processing of this program by the PTS system established that there are 56 PTS-segments.

After the set of PTS-segments are linked together to form segments as previously defined, they were found to number 36. These can be represented as a directed graph as shown in Figure 12.

As noted in previous discussions, the number of possible paths through a system may be fairly large. But as also mentioned before, it has been found "experimentally" that many of the potential paths are not realizable. To this point, in Figure 13, there is shown the result of using 20 cases consisting of coefficients (48) chosen from a β-distribution on the logarithm and positions chosen by a β-distribution on each coordinate in the unit square. This result is interesting in at least two respects. First, the paths shown account for only 10 out of 31 of the segments. But, more importantly, the response is perfectly consistent: every one of the 20 random test cases which were used, exercised all three of the branches numbered 8, 9, and 10 of Figure 13. Thus the randomly selected cases exhibited a remarkable consistency.

Although 20 is a small sample size it should be stated that in separate runs, distributions other than the β were employed and identical results were obtained. In a certain sense, this rather simple program has been exhaustively tested by just one sample. The sense, of course, is that all the branches ever executed by random selection is accomplished on the first "draw".

The algorithm employed so successfully before is of no value in estimating the number of execution sequences. An alternative is mentioned below.

```
      SUBROUTINE CURVTR ( V,U,W,CV,DC )                                 CURV0010
C       THIS ROUTINE WILL FIND THE CURVATURES OF A LINE DEFINED BY THE  CURV0020
C       INTERSECTION OF EACH OF THE THREE ORTHOGONAL PLANES WITH THE    CURV0030
C       PATCH (V) AT THE POINT (U,W), DIRECTION COSINES OF THE          CURV0040
C       SURFACE NORMALS AT THAT POINT ARE PLACED IN (DC).               CURV0050
CAUTION - THIS ROUTINE WILL NOT COMPUTE CURVATURE AT THE DEGENERATE     CURV0060
C          EDGE OF A PATCH                                              CURV0070
C           ARGUMENT DESCRIPTION                                        CURV0080
C           V(16,3) INPUT, PATCH COEFFICIENTS IN ALGEBRAIC FORM         CURV0090
C           U        INPUT, PARAMETRIC VARIABLE U                       CURV0100
C           W        INPUT, PARAMETRIC VARIABLE W                       CURV0110
C           CV(1)   OUTPUT, CURVATURE IN Y-Z PLANE                      CURV0120
C           CV(2)   OUTPUT, CURVATURE IN X-Z PLANE                      CURV0130
C           CV(3)   OUTPUT, CURVATURE IN X-Y PLANE                      CURV0140
C           DC(1)   OUTPUT, X DIRECTION COSINE                          CURV0150
C           DC(2)   OUTPUT, Y DIRECTION COSINE                          CURV0160
C           DC(3)   OUTPUT, Z DIRECTION COSINE                          CURV0170
C       SUBROUTINES CALLED:  PLACE                                      CURV0180
      DIMENSION B(16),C(3),CV(3),DC(3),V(16,3),VU(3),VW(3),VUU(3),VWW(3)CURV0200
     1,VUW(3),A1(3,2),A2(3,2)                                           CURV0210
      EQUIVALENCE (VU(1),A1(1,1)),(VW(1),A1(1,2)),(VUU(1),A2(1,1)),     CURV0220
     1(VWW(1),A2(1,2))                                                  CURV0230
      U3 = 3.00*U                                                       CURV0240
      U6 = U3 + U3                                                      CURV0250
      W3 = 3.00*W                                                       CURV0260
      W6 = W3 + W3                                                      CURV0270
      DO 10 I = 1,3                                                     CURV0280
      CALL PLACE ( V(1,I),B,0 )                                         CURV0290
      B2 = B(2) + B(2)                                                  CURV0300
      B6 = B(6) + B(6)                                                  CURV0310
      B10 = B(10) + B(10)                                               CURV0320
      B14 = B(14) + B(14)                                               CURV0330
      VUW(I) = ((B(1)*U3+B2)*U+B(3))*W3*W*((B(5)*U3+B6)*U+B(7))*(W+W)   CURV0340
     1 +(B(9)*U3+B10)*U+B(11)                                           CURV0350
      VUU(I)=(((B(1)*U6+B2)*W+B(5)*U6+B6)*W+B(9)*U6+B10)*W+B(13)*B6+B14 CURV0360
      VU(I) = ((((B(1)*U3+B2)*U+B(3))*W+(B(5)*U3+B6)*U+B(7))*W+(B(9)*U3 CURV0370
     1*B10)*U+B(11))*W+(B(13)*U3+B14)*U+B(15)                           CURV0380
      B2 = B(5) + B(5)                                                  CURV0390
      B10 = B(7) + B(7)                                                 CURV0400
      B14 = B(8) + B(8)                                                 CURV0410
      VWW(I)=(((B(1)*W6+B2)*U+B(2)*W6+B6)*U+B(3)*W6+B10)*U_B(4)*W6+B14  CURV0420
      VW(I) = ((((B(1)*W3+B2)*W+B(9))*U+(B(2)*W3+B6)*W+B(10))*U+(B(3)*  CURV0430
     1W3+B10)*W+B(11))*U+(B(4)*W3+B14)*W+B(12)                          CURV0440
   10 CONTINUE                                                          CURV0450
      B(1) = VU(1)*VU(1) + VU(2)*VU(2) + VU(3)*VU(3)                    CURV0460
      B(2) = VU(1)*VW(1) + VU(2)*VW(2) + VU(3)*VW(3)                    CURV0470
      B(3) = VW(1)*VW(1) + VW(2)*VW(2) +VW(3)*VW(3)                     CURV0480
      U3 = 1.00 / ( B(1)*B(3) - B(2)*B(2) )                             CURV0490
      U6 = SQRT ( U3 )                                                  CURV0500
      B(4) = U6*( VU(2)*VW(3) - VU(3)*VW(2) )                           CURV0510
      B(5) = U6*( VU(3)*VW(1) - VU(1)*VW(3) )                           CURV0520
      B(6) = U6*( VU(1)*VW(2) - VU(2)*VW(1) )                           CURV0530
      B(7) = VUU(1)*B(4) + VUU(2)*B(5) + VUU(3)*B(6)                    CURV0540
      B(8) = VUW(1)*B(4) + VUW(2)*B(5) + VUW(3)*B(6)                    CURV0550
      B(9) = VWW(1)*B(4) + VWW(2)*B(5) + VWW(3)*B(6)                    CURV0560
```

Figure 11. Listing of CURVTR Subroutine (Page 1 of 3)

```
      DO 70 I = 1,3                                              CURV0570  31
      CV(I) = 0.00                                               CURV0580
      B(I+9) = U3*( B(3)*VU(I) - B(2)*VW(I) )                    CURV0590
      B(I+12) = U3*( B(1)*VW(I) - B(2)*VU(I) )                   CURV0600
      IF (  ABS( B(I+3) ) ,LT, ,99999D0 ) GO TO 70               CURV0610  35   36
      J = 0                                                      CURV0620
      IF ( B(7) .NE. 0.00) GO TO 20                              CURV0640  38   39
      J = 1                                                      CURV0640
      B(16) = 1.00 / ( B(1)* SQRT( B(1) ) )                      CURV0650  41
      GO TO 30                                                   CURV0660
   20 IF ( B(9) .NE. 0.00 ) GO TO 70                             CURV0670  43   44
      J = 2                                                      CURV0680
      B(16) = 1.00 / ( B(3)* SQRT( B(3) ) )                      CURV0690  46
   30 IF ( I - 2 ) 40 , 50 , 60                                  CURV0700
   40 CV(1) = ( A1(3,J)*A2(2,J) = A1(2,J)*A2(3,J) )*B(16)        CURV0710
      GO TO 70                                                   CURV0720
   50 CV(2) = ( A1(3,J)*A2(1,J) = A1(1,J)*A2(3,J) )*B(16)        CURV0730
      GO TO 70                                                   CURV0740  51
   60 CV(3) = ( A1(2,J)*A2(1,J) = A1(1,J)*A2(2,J) )*B(16)        CURV0750
   70 CONTINUE                                                   CURV0760
      DO 170 I = 1,3                                             CURV0770
      W6 = B(I+3)                                                CURV0780
      B(I+3) = 0.00                                              CURV0790  56
      U6 = B(4)*B(4) + B(5)*B(5) + B(6)*B(6)                     CURV0800
      VUW(I) = SQRT( U6 )                                        CURV0810
      B(I+3) = W6                                                CURV0820
      IF ( U6 .EQ. 0.00 ) GO TO 110                              CURV0830  60   61
      C(I) = 0.00                                                CURV0840
      IF ( I = 2 ) 80 , 90 , 100                                 CURV0850
   80 C(2) = -B(6)                                               CURV0860
      C(3) = B(5)                                                CURV0870
      GO TO 150                                                  CURV0880  66
   90 C(1) = B(6)                                                CURV0890
      C(3) = -B(4)                                               CURV0900
      GO TO 150                                                  CURV0910
  100 C(1) = -B(5)                                               CURV0920
      C(2) = B(4)                                                CURV0930  71
      GO TO 150                                                  CURV0940
  110 U3 = 1.00                                                  CURV0950
      W3 = 1.00                                                  CURV0960
      IF ( VW(I) .EQ, 0.00 ) GO TO 120                           CURV0970  75   76
      W3 = -VU(I) / VW(I)                                        CURV0980
      GO TO 140                                                  CURV0990
  120 IF ( VU(I) .EQ, 0.00 ) GO TO 130                           CURV1000  79   80
      U3 = 0.00                                                  CURV1010  81
      GO TO 140                                                  CURV1020
  130 IF ( ABS(VU(1))+ ABS(VU(2))+ ABS(VU(3)) .EQ, 0.00 ) GO TO 140  CURV1030  83   84
      W3 = 0.00                                                  CURV1040
  140 C(1) = U3*VU(1) + W3*VW(1)                                 CURV1050  86
      C(2) = U3*VU(2) + W3*VW(2)                                 CURV1060
      C(3) = U3*VU(3) + W3*VW(3)                                 CURV1070
      U6 = C(1)*C(1) + C(2)*C(2) + C(3)*C(3)                     CURV1080
      IF ( U6 .EQ. 0.00 ) GO TO 160                              CURV1090  90   91
  150 U6 = 1.00 / SQRT( U6 )                                     CURV1100
  160 C(1) = C(1)*U6                                             CURV1110
      C(2) = C(2)*U6                                             CURV1120
```

Figure 11. Listing of CURVTR Subroutine (Page 2 of 3)

```
      C(3) = C(3)*U6                                            CURV1130
      VUU(I) = C(1)*B(10) + C(2)*B(11) + C(3)*B(12)             CURV1140   96
      VWW(I) = C(1)*B(13) + C(2)*B(14) + C(3)*B(15)             CURV1150
  170 CONTINUE                                                  CURV1160
      DO 190 I = 1,3                                            CURV1170
      DC(I) = B(I+3)                                            CURV1180
      IF ( CV(I) .NE. 0.00 ) GO TO 190                          CURV1190  101  102
      IF ( ABS( VUU(I) ) .LT. .1E-30 ) VUU(I) = 0.00            CURV1200  103  104
      IF ( ABS( VWW(I) ) .LT. .1E-30 ) VWW(I) = 0.00            CURV1210  105  106
      VU(1) = VUU(I)*VUU(I)                                     CURV1220
      VU(2) = ( VUU(I)*VWW(I) ) + ( VUU(I)*VWW(I) )             CURV1230
      VU(3) = VWW(I)*VWW(I)                                     CURV1240
      VW(I) = B(7)*VU(1) + B(8)*VU(2) + B(9)*VU(3)              CURV1250
      IF ( VUW(I) .EQ. 0.00 ) GO TO 180                         CURV1260  111  112
      Cr(I) = VW(I) / VUW(I)                                    CURV1270
      GO TO 190                                                 CURV1280
  180 CV(I) = 1.E40                                             CURV1300
  190 CONTINUE                                                  CURV1300
      RETURN                                                    CURV1310
      END                                                       CURV1320
```

Figure 11. Listing of CURVTR Subroutine (Page 3 of 3)

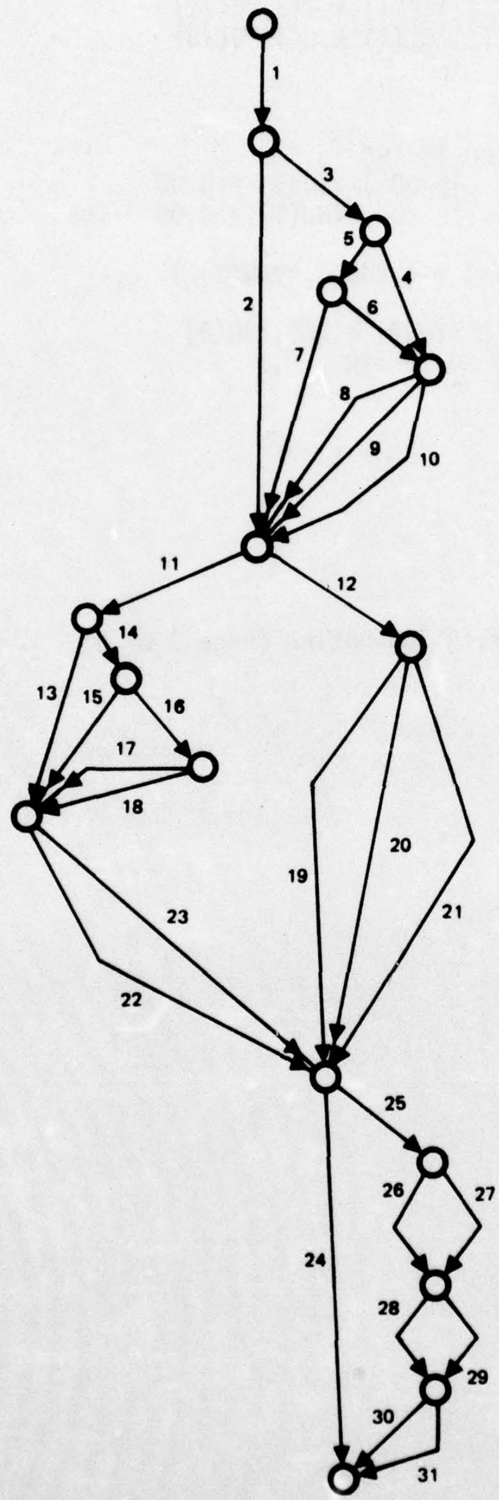

Figure 12. Segmented CURVTR Program

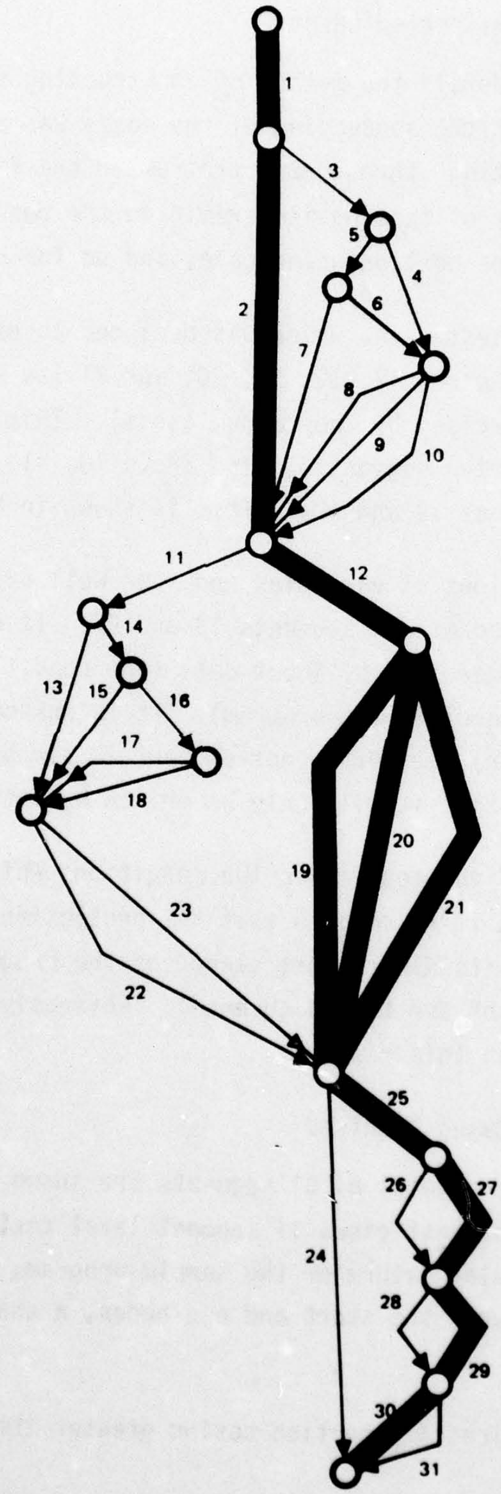

Figure 13. Segments Driven by Random Inputs

AD-A035 585 MCDONNELL DOUGLAS ASTRONAUTICS CO HUNTINGTON BEACH CALIF F/G 9/2
QUANTITATIVE METHODS FOR SOFTWARE RELIABILITY MEASUREMENTS.(U)
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4. Testing by Constructed Cases

Without discussing in detail the method of constructing the special test cases, it is noted that the sequencing of the cases was done in the most natural way, that is, by "painting" through the program to the first untested branch, inventing a way of testing it, examining the result of the test case, retracing through to the next untested case, and so forth.

The first constructed test case, which was designed to exercise segment number 3, also exercised segments 5, 7, 12, 19, 20, and 21 (as well as 25, 27, 29, and 31 which were also exercised by the random cases). This is shown in Figure 14. The test designed to drive segment 4, also drove 10, 11, 14, 16, 17, 23, 25, 26, 23 and 30 (as well as 19 and 20). This is shown in Figure 15.

After obvious permutations of variables and four well designed cases all of the segments were tested except segments 13 and 27. It can be shown that segment 13 cannot be exercised by input data (the condition which is necessary cannot exist at this level of the program). It is unknown (and not worth the effort to establish) whether or not segment 27 can be driven by data, but is is safe to say that it will only be driven by data on a knife-edge.

In the construction of the test cases the conditions which had to be met were very severe*, and it is doubted that the protection afforded by the programming was worth its direct cost (labor of the programmer) or the indirect cost (a "crash" due to its absence). Obviously each program requires a separate judgement in this respect.

5. Number Test Cases Required

In the sample problem, a total of 31 segments are shown and this is an upper bound for the number of test cases if segment level testing is required. Because of the particular nature of the sample program, "closing" as it does at two nodes between the start and end nodes, a sharper upper bound can be obtained.

*one test branch requires a direction cosine greater than .99999 (instruction 0610).

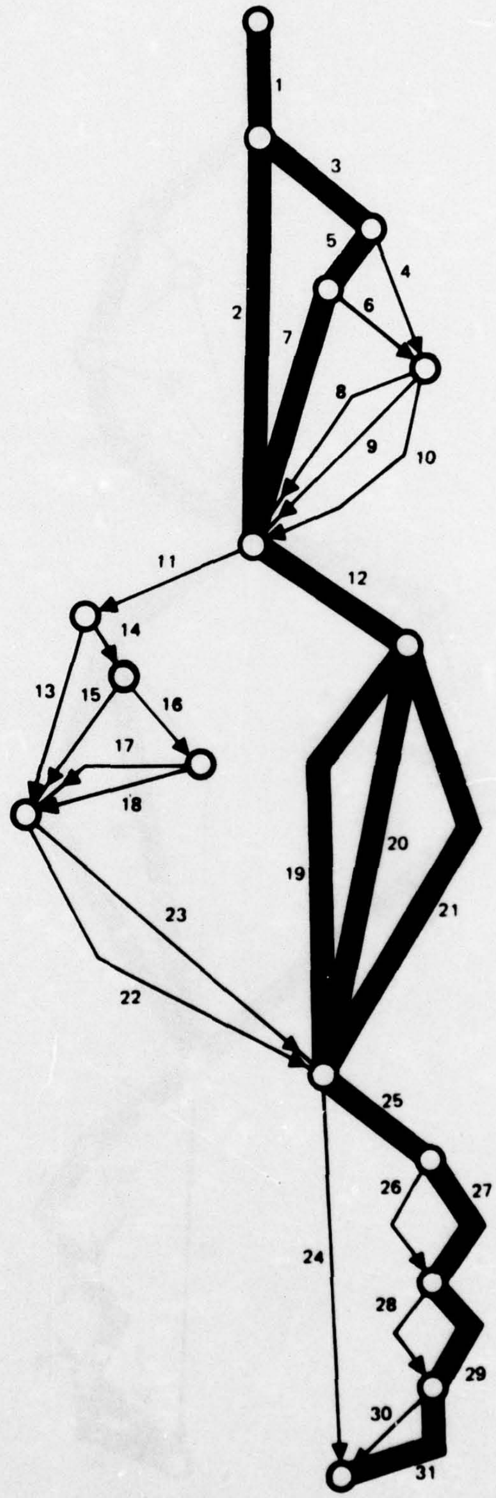

Figure 14. Constructed Case I

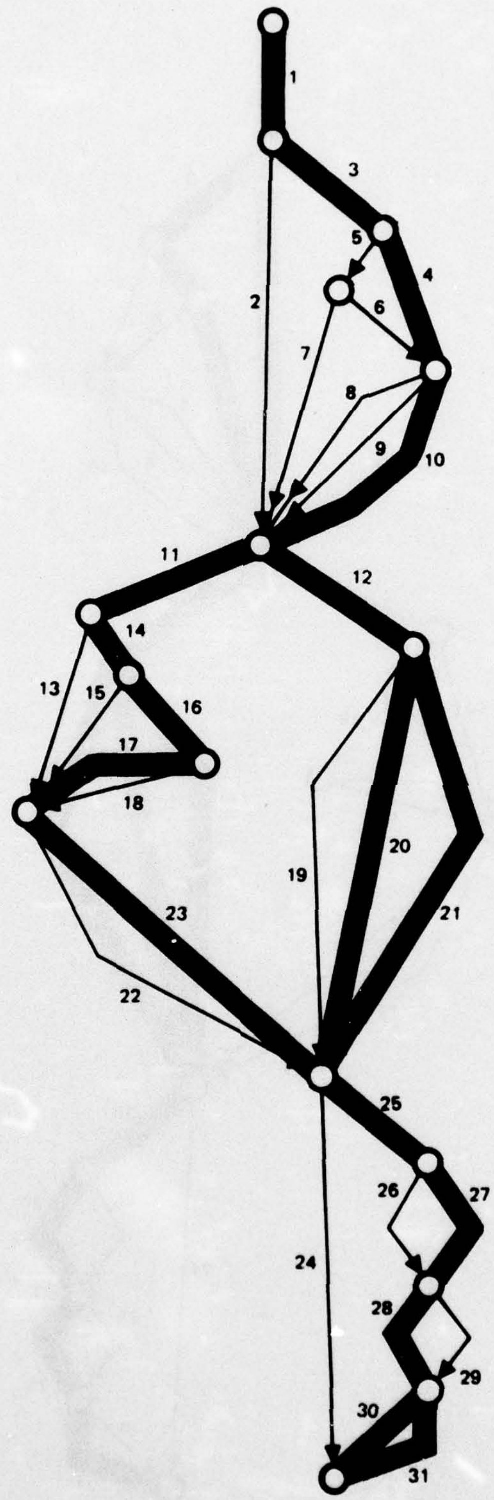

Figure 15. Constructed Case II

Inspection shows that the bottom cluster can be exercised completely by at most 5 test cases. Any path from the top node in the bottom cluster starting with 25 will drive three more segments. There are then only four other segments to drive (in actuality it was found that 3 cases covered all of the segments in the bottom cluster). A generous upper bound to the cases required to the coverage (at the segment level testing) of both the middle and bottom clusters can be found by adding to those segments in the middle cluster which were driven in covering the segments in the bottom cluster (and are therefore already covered and not to be doubly counted), those segments still uncovered. In the specific example this turns out to be 3, but in the general case where no other information would be available, it would only be possible to assume that one of the middle branches were used, so that the number which might be used - this would assume that all of the segments driven in the bottom cluster can be driven by data which drives the same branch in the middle cluster - would be one less than the number of segments in the middle branch minus the number of segments in the shortest branch in the middle cluster. This (pessimistic) estimate in the sample program would be 10. But once data has been used and the segments in the bottom cluster are driven, it turns out that there are only three segments in the middle left uncovered. It also turns out that all of the segments in the top cluster are exercised when the bottom two clusters, are, but without the knowledge so obtained, it would be necessary to use the total count 10 (or 9 since segment 1 is always driven), since the shortest branch is of length 1.

Thus a generous upper limit for the test cases, in the absence of any other knowledge would be (in this special kind of case), 25. On the other hand if the estimate were sequentially formed on the basis of the results obtained by the constructed cases, they would be 21, 10, 6, 5, 5 (a fast converging sequence).

V. DEVELOPMENT OF DETECTION RATE MODELS

A. The De-Eutrophication Process

This model is described in an overview manner in Section III.

The data for analysis consist of the sequence of times between errors: $X_1, X_2, \ldots, X_n$, as shown in Figure 4 (Section III).

1. Maximum Likelihood Estimates of Parameters

Estimation of the parameters N and ϕ can be made by application of the maximum likelihood principle. Under the assumption that there is a uniform failure rate between errors, the density for X_i is given by

$$p(X_i) = \phi[N-(i-1)] \exp\left\{-\ \phi[N-(i-1)]X_i\right\} \tag{13}$$

and the likelihood function is

$$L(X_1,X_2,\ldots,X_n) = \prod_{i=1}^{n} \phi[N-(i-1)] \exp\left\{-\phi[N-(i-1)]X_i\right\} \tag{14}$$

It is more convenient to maximize the natural logarithm of the likelihood:

$$\log_e L = \sum_{i=1}^{n} \log[N-(i-1)]\phi - \sum_{i=1}^{n} [N-(i-1)]\phi X_i \tag{15}$$

$$= \sum_{i=1}^{n} \log[N-(i-1)] + n \log \phi - \sum_{i=1}^{n} [N-(i-1)]\phi X_i \tag{15a}$$

Taking partials and setting to zero

$$\frac{\partial \log L}{\partial N} = \sum_{i=1}^{n} \frac{1}{N-(i-1)} - \sum_{i=1}^{n} \phi X_i = 0 \tag{16}$$

$$\frac{\partial \log L}{\partial \phi} = \frac{n}{\phi} - \sum_{i=1}^{n} (N-(i-1))X_i = 0 \tag{17}$$

Letting $\sum X_i = T$, Eq. 17 can be solved for ϕ to yield

$$\phi = \frac{n}{NT - \sum(i-1)X_i} \tag{18}$$

and Eq. 16 becomes

$$\sum_{i=1}^{n} \frac{1}{(N-(i-1))} = \frac{nT}{NT - \sum (i-1)X_i} = \frac{n}{n - \frac{1}{T} \sum_{i=1}^{n} (i-1)X_i} \tag{19}$$

Equation 19 is free of ϕ and presents the key equation to solve. The two data-derived statistics are $T = \sum X_i$ and $\sum(i-1)X_i$. Knowing these we can solve Eq. 19 for N, the initial error content.

Solving this for N, and calling the result $\hat{N}$, we can obtain the estimate for

$$\phi = \frac{n}{\hat{N}T - \sum(i-1)X_i} \tag{20}$$

The preceding development is essentially a mathematical exercise and does not explicitly employ properties of the Poisson process; accordingly, the following heuristic argument is given for the genesis of Eqs. (16) and (17).

The mean, or expected value, of a random variable having an exponential distribution, $\lambda e^{-\lambda x}$, is equal to λ^{-1} and in the particular terminology of the Poisson process, this is called the mean-time-between-failures. Thus, a reasonable "representative" for the variable X_1, whose distribution is exponential with parameter $N\phi$, is $\frac{1}{N\phi}$; similarly the quantity $\frac{1}{N\phi}$ "represents" X_2, and so forth. Thus summing over the n variables, there results

$$\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} \frac{1}{(N-i-1))\phi}$$

which is essentially Eq. 16. With the expectation operator applied to the sum this equation would be precise.

For a Poisson process with a uniform failure rate, θ, a reasonable estimate for θ would be the ratio formed by dividing the number of failures observed by the time of observation. Another way of looking at this, and a way which

permits generalization, is to estimate the number of failures occurring in a given period by integrating the (constant) failure rate over that period. This same procedure will apply to a variable failure rate, and in the particular process of the model, the integration becomes very simple. Performing the integration and setting it equal to n produces the equation

$$n = N\phi X_1 + (N-1)\phi X_2 + \ldots + (N-(n-1))\phi X_n$$

or

$$\frac{n}{\phi} = \sum_{i=1}^{n} (N-(i-1))X_i$$

which is seen to be the same as Eq. 17.

a. Estimate of MTTF

From general considerations, an estimate for other functions of the two variables N and ϕ can be obtained by substitution of their estimated value into the functional relation.

In particular, the estimate of the MTTF can be obtained by taking the reciprocal of the failure rate at the end of the observation period. In the present instance, the next error is the (n+1)st error and the estimate for the MTTF is

$$\hat{M}_1 = [\hat{N}-n]\hat{\phi}^{-1} \qquad (21)$$

where we have used a subscript to distinguish between the estimates which different models provide.

b. Estimate of Purification Percentage

For comparison purposes, the degree of purification which has been achieved through testing can be used. It is simply the ratio of the difference between the initial and final failure rate and the initial failure rate. In the present instance this is

$$P_1 = 100.\frac{\hat{N}\hat{\phi} - [\hat{N}-n]\hat{\phi}}{\hat{N}\hat{\phi}} = 100.\frac{n}{\hat{N}} \qquad (22)$$

where we, again, use a subscript to denote the model associated with the estimate.

c. Variances/Covariances of Estimates

The general properties of maximum likelihood estimates can be used in a purely formal way to derive some measure of the variability in the estimates. This point must be emphasized since it is manifest that the use of asymptotic formulas (involving large sample sizes) on samples which are fundamentally limited to be finite (there can be no larger samples than there are errors) can result only in caution-laden approximations. Nonetheless, the experiences which have been gained using the models seem to indicate that these approximations for the variances are generally much too large.

The basis for the development of the large sample estimates is a theorem due to R. A. Fisher which states that under certain "general conditions", which have to do with the boundedness of the first three derivatives of the likelihood, the variance and covariances of the estimates are given by the inverse of a matrix formed from the mathematical expectation of second partial derivatives.

Explicitly the matrix A_{ij} (which is to be inverted) employed in the estimation of several parameters $(\theta_1, \theta_2, \ldots, \theta_n)$ has the terms

$$A_{ij} = -E \left\{ \frac{\partial^2 \log L}{\partial \theta_i \partial \theta_j} \right\} \quad . \tag{23}$$

From Eqs. (16) and (17) above

$$\frac{\partial^2 L}{\partial N^2} = - \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} \tag{24}$$

$$\frac{\partial^2 L}{\partial N \partial \phi} = \frac{\partial^2 L}{\partial \phi \partial N} = - \sum_{i=1}^{n} X_i \tag{25}$$

$$\frac{\partial^2 L}{\partial \phi^2} = - \frac{n}{\phi^2} \tag{26}$$

And since

$$E(X_i) = \frac{1}{(N-i+1)\phi}$$

the matrix elements become:

$$A_{11} = \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} \tag{27}$$

$$A_{12} = A_{21} = \sum_{i=1}^{n} \frac{1}{(N-i+1)\phi} \tag{28}$$

$$A_{22} = \frac{n}{\phi^2} \tag{29}$$

where for evaluation in practical situations, the values of $\hat{N}$ and $\hat{\phi}$ (the estimates based on the data) are used. The determinant (denoted Det_1) of the A-matrix is

$$Det_1 = A_{11}A_{22}-A_{12}A_{21} = \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} \cdot \frac{n}{\phi^2} - T^2 \tag{30}$$

where we have used the fact that "on the average"

$$\sum_{i=1}^{n} \frac{1}{(N-i+1)\phi} = T, \text{ the total observation time.}$$

Hence

$$Var(\hat{N}) = \frac{n}{\phi^2} \cdot \frac{1}{Det_1} \tag{31}$$

$$Var(\hat{\phi}) = \sum_{i=1}^{n} \frac{1}{N-i+1}^{2} \cdot \frac{1}{Det_1} \tag{32}$$

$$Covar(\hat{N},\hat{\phi}) = -\frac{T}{Det_1} \tag{33}$$

Since for a fixed sample size n, the solutions for N and ϕ by means of Eqs. (18) and (19), depend only on the ratio $R = \frac{\Sigma(i-1)X_i}{\Sigma X_i}$, so also can

the variance and covariance be determined from R. This is done in the subsequent section.

2. Explanation and Development of Appendix II

The solutions to the maximum likelihood equations and the subsequent computation of the MTTF and other derived measures are difficult to wring out. A material assist is provided by the tables which form Appendix II.

Since for a fixed sample size n, the solutions for N and ϕ by means of Eqs. 18 and 19 depend only on the ratio $R = \frac{\Sigma(i-1)X_i}{\Sigma X_i}$, it is possible to tabulate solutions as a function of the ratio. With the so-determined solutions, the (estimated) variances and covariances, and the MTTF can be obtained. Such a table can be computed for each integer n.

In order to tabulate the parameters for an arbitrary realization of the process, it is necessary that the scale for time be normalized. Since the total observation time, T, is assumed recorded by the data collection process, it is a natural scale factor to use. It must be pointed out however, that this time is a random variable; although it is treated as if it were a constant, this is a purely pragmatic interpretation. A reasonable interpretation which can be made is that the results which are recorded are conditional on the observed time.

Given the ratio R, the MLEs become

$$\sum_{i=1}^{n} \frac{1}{N-(i-1)} = \frac{n}{N-R} \tag{34}$$

and

$$\phi T = \frac{n}{N-R} \tag{35}$$

Equation (34) can be solved essentially by trial and error. Once N is established, the quantity ϕT can be obtained from Eq. (35). The quality ϕT is entered in column 3 of the sample table, Table XI.

The variance of N can be obtained in the following way.

Table XI

Sample Table

(n = 26)

Ratio	Error Content	(PHI)T	DEVN	DEVØ (Normed)	COVAR (Normed)	MTTF (Normed)
14.0	51.19	.6991	35.88	.6883	-24.2005	.0568
14.2	46.94	.7942	27.74	.6907	-18.6666	.0601
14.4	43.62	.8899	22.04	.6936	-14.7968	.0638
14.6	40.95	.9866	17.87	.6966	-11.9618	.0678
14.8	38.78	1.0842	14.75	.7001	-9.8426	.0722
15.0	36.98	1.1826	12.36	.7041	-8.2155	.0770
15.2	35.47	1.2824	10.47	.7083	-6.9311	.0823
15.4	34.19	1.3836	8.95	.7129	-5.9020	.0882
15.6	33.10	1.4857	7.73	.7181	-5.0736	.0948
15.8	32.15	1.5898	6.72	.7235	-4.3850	.1022
16.0	31.34	1.6953	5.88	.7296	-3.8158	.1105
16.2	30.62	1.8027	5.17	.7361	-3.3350	.1200
16.4	30.00	1.9121	4.56	.7432	-2.9274	.1308
16.6	29.45	2.0236	4.05	.7508	-2.5795	.1433
16.8	28.96	2.1377	3.60	.7591	-2.2784	.1579
17.0	28.53	2.2541	3.21	.7681	-2.0185	.1750
17.2	28.15	2.3737	2.87	.7777	-1.7909	.1956
17.4	27.82	2.4959	2.58	.7883	-1.5935	.2205
17.6	27.52	2.6222	2.32	.7995	-1.4173	.2516
17.8	27.25	2.7519	2.08	.8119	-1.2630	.2912
18.0	27.01	2.8862	1.87	.8251	-1.1250	.3436
18.2	26.80	3.0247	1.69	.8396	-1.0035	.4154
18.4	26.61	3.1680	1.52	.8556	-.8964	.5200

Column 1 is the ratio $\Sigma(i-1)X_i / \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to botain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and Ø: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

By Eq. (31)

$$\mathrm{Var}(N) = \frac{n}{\phi^2 \mathrm{Det}_1} \tag{36}$$

but using the substitution

$$S_2 = \sum_{i=1}^{n} \frac{1}{(N-i+1)^2} \tag{37}$$

the determinant of Eq. (30) can be expressed as

$$\mathrm{Det}_1 = \frac{nS_2}{\phi^2} - T^2$$

or

$$\phi^2 \mathrm{Det}_1 = nS_2 - (\phi T)^2$$

Hence the denominator of Eq. 36 can be evaluated using the estimates ϕT and N.

Hence

$$\mathrm{Var}(N) = \frac{n}{nS_2 - (\phi T)^2} \tag{38}$$

The standard deviation is the more useful measure and is obtained by taking the square root of Var (N). This is entered in column 4 of the sample table.

The variance of ϕT is obtained in much the same way: Det_1 is evaluated as before, and with S_2 as defined,

$$\mathrm{Var}(\phi T) = T^2 \frac{S_2}{\mathrm{Det}_1} = \frac{S_2 T^2}{S_2 \frac{n}{\phi^2} - T^2}$$

$$= \frac{S_2(\phi T)^2}{nS_2 - (\phi T)^2} \tag{39}$$

D can be eliminated from both equations to leave a single equation.

$$\frac{\sum_{i=1}^{n} i\, k^{i-1} X_i}{\sum k^{i-1} X_i} = \frac{n+1}{2} \qquad (45)$$

a. Estimate of MTTF

For this model the MTTF at the end of the test, where n errors have been detected, is given by the reciprocal of the failure rate for the n+1st error:

Thus

$$MTTF_2 = \left(\hat{D}\hat{k}^n\right)^{-1} \qquad (46)$$

b. Estimate of Purification Percent

For comparison purposes among models the degree of achievement of the "ultimate" is measured, as before by the ratio formed by dividing the difference between the initial and final failure rate by the initial failure rate.

Thus, in percent

$$\hat{P}_2 = (1-\hat{k}^n)\ (100) \qquad (47)$$

c. Variances/Covariances of Estimates

The variance and covariances for this process are found by the same procedure used above. Distinct from the case above, however, this process has an infinite number of errors, and so the sample size can become large, and the asymptotic formulas can be applied without apology.

Directly by differentiation of the likelihood given in Eq. (42),

$$\frac{\delta^2 \log L}{\delta D^2} = -\frac{n}{D^2} \qquad (48)$$

$$\frac{\delta^2 \log L}{\delta D \delta k} = \frac{\delta^2 \log L}{\delta k \delta D} = - \sum_{i=1}^{n} (i-1) k^{i-2} X_i \tag{49}$$

$$\frac{\delta^2 \log L}{\delta k^2} = p \frac{1}{k^2} \sum_{i=1}^{n} (i-1) - D \sum_{i=1}^{n} (i-1)(i-2) k^{i-3} X_i \tag{50}$$

Since $E(X_i) = \frac{1}{Dk^{i-1}}$, the associated A-matrix elements are

$$A_{11} = \frac{n}{D^2} \tag{51}$$

$$A_{12} = A_{21} = \frac{1}{Dk} \sum_{i=1}^{n} (i-1) = \frac{1}{Dk} \frac{n(n-1)}{2} \tag{52}$$

$$A_{22} = \frac{1}{k^2} \sum_{i=1}^{n} (i-1) + \frac{1}{k^2} \sum_{i=1}^{n} (i-1)(i-2) \tag{53}$$

$$= \frac{1}{k^2} \sum_{i=1}^{n} (i-1)^2 = \frac{1}{6k^2} n(n-1)(2n-1) \tag{54}$$

Using Det_2 to represent the determinant of the A-matrix, we obtain after simple reduction:

$$Det_2 = \frac{1}{D^2 k^2} \cdot \frac{n^2(n^2-1)}{12} \tag{55}$$

Thus, the variances and covariances are

$$\text{Var } \hat{D} = D^2 \frac{2(2n-1)}{n(n+1)} \tag{56}$$

$$\text{Var } \hat{k} = k^2 \frac{12}{n(n^2-1)} \tag{57}$$

$$\text{Covar } (\hat{D},\hat{k}) = -Dk \frac{6}{n(n+1)} \tag{58}$$

In the limit these variances tend to zero. On the other hand, it will be noted that the correlation coefficient between the estimates is quite high (in absolute value):

$$\rho = -\sqrt{\frac{3}{2}\,\frac{(n-1)}{(2n-1)}} \tag{59}$$

which is in excess of 0.85.

The estimate for the MTTF which has the character of the maximum likelihood estimates is given by Eq. (27) as

$$\hat{M}_2 = \frac{1}{\hat{D}\hat{k}^n}$$

where the subscript 2 denotes the estimate for this process. The asymptotic approximations can be employed in another reasonable approximation in order to derive a measure of the variation in the estimate of the MTTF. By differentiation taking the total differential and expectations it is seen that

$$\mathrm{Var}\, M_2 = \frac{1}{D^2 k^{2n}}\,\mathrm{Var}(D) + \frac{2n}{D^3 k^{2n+1}}\,\mathrm{Covar}\,(D,k) +$$

$$\frac{n^2}{D^2 k^{2n+2}}\,\mathrm{Var}(k) \tag{60}$$

where, again the estimates would be used as proxies for the (unknown) parameters.

2. Explanation of Appendix III

After elimination of one of the parameters from the pair of maximum likelihood equations, there results a polynomial (in k). This polynomial has the (random) separation times $x_1, x_2, \ldots, x_n$ for coefficients. Of course, it is not possible to formulate the equation until a realization of the process has occurred; furthermore, the relation between D and k shown in Eq. 45, involves evaluation of this polynomial. Consequently, there is no way to construct tables for this process.

Appendix III provides a convenient summary of the formulas which are associated with that process.

C. Hybrid Geometric/Poisson Process

Analysis of this model which is described in preliminary form in Section III, follows that of the two de-eutrophication models. Because some results have been produced which have not previously published in open literature a more extensive development is made as well as a description of an application.

1. Maximum Likelihood Estimates of Parameters

The likelihood function for the error separation times $X_i = T_i - T_{i-1}$, for $i=1,2,\ldots,n$, and $T_0=0$, is:

$$L = \prod_{i=1}^{n} \left(Dk^{i-1}+\theta\right) \exp\left\{-\left(Dk^{i-1}+\theta\right) X_i\right\}.$$

The maximum likelihood equations are obtained by partial differentiation of the logarithm of the likelihood function.These are

$$\sum_{i=1}^{n} \frac{k^{i-1}}{Dk^{i-1}+\theta} - \sum_{i=1}^{n} k^{i-1}X_i = 0 \tag{61}$$

$$\sum_{i=1}^{n} \frac{(i-1)k^{i-2}}{Dk^{i-1}+\theta} - \sum_{i=1}^{n} (i-1)k^{i-2}X_i = 0 \tag{62}$$

$$\sum_{i=1}^{n} \frac{1}{Dk^{i-1}+\theta} - \sum X_i = 0 \tag{63}$$

By simple manipulation on Eq. 61 it can be converted to

$$n - \theta \sum_{i=1}^{n} \frac{1}{Dk^{i-1}+\theta} = \sum Dk^{i-1}X_i \tag{64}$$

and, from Eq. 63, using $T = \sum_{i=1}^{n} X_i$, the explicit relation

$$\theta = \frac{n - \sum_{i=1}^{n} Dk^{i-1}X_i}{T}, \tag{65}$$

can be obtained.

Since the second term of the numerator represents the integral of the failure rate out to time T, and so is an estimate of the number of non-repeatable errors, the numerator represents the number of "true" Poisson-like errors. Consequently, the quotient of this number divided by time is similar to the commonly employed estimate.

The Eqs. 61, 62 and 63 constitute a set of non-linear equations which can be solved by either the Newton method by iteration with the Jacobian, or by the method,due to K. M. Brown [17]. The latter method is used in the illustrative example following.

Although the Brown method is used, the possibility of a solution is determined by the evaluation of Jacobian (determinant) associated with the three equations. If $F_1(D,k,\theta)$, $F_2,(D,k,\theta)$, and $F_3(D,k,\theta)$ are used to denote the left sides of 61, 62 and 63, respectively, and replace D, k, and θ by θ_1, θ_2, θ_3, then the Jacobian entries are obtained by differentiation. Thus, generically we have

$$J_{ij} = \frac{\delta F_i}{\delta \theta_j}$$

and the particular entries are

$$J_{11} = -\sum_{i=1}^{n} \frac{k^{2i-2}}{(Dk^{i-1}+\theta)^2} \qquad J_{31} = -\sum_{i=1}^{n} \frac{k^{i-1}}{(Dk^{i-1}+\theta)^2}$$

$$J_{12} = \sum_{i=1}^{n} \frac{(i-1)k^{2i-3}}{(Dk^{i-1}+\theta)^2} \qquad J_{32} = -\sum_{i=1}^{n} \frac{(i-1)k^{i-2}}{(Dk^{i-1}+\theta)^2}$$

$$J_{13} = -\sum_{i=1}^{n} \frac{k^{i-1}}{(Dk^{i-1}+\theta)} \qquad J_{33} = -\sum_{i=1}^{n} \frac{1}{(Dk^{i-1}+\theta)^2}$$

$$J_{21} = -\sum_{i=1}^{n} \frac{(Dk^{i-1}+\theta)(i-1)k^{i-2}-k^{i-1}D(i-1)k^{i-2}}{(Dk^{i-1}+\theta)^2} - \sum_{i=1}^{n} (i-1)k^{i-2}X_i$$

$$J_{22} = -\sum_{i=1}^{n} \frac{(Dk^{i-1}+\theta)(i-1)(i-2)k^{i-3}-(i-1)k^{i-2}D(i-1)k^{i-2}}{Dk^{i-1}+\theta)^2} - \sum_{i=1}^{n}(i-1)k^{i-2}X_i$$

$$J_{23} = -\sum_{i=1}^{n} \frac{D(i-1)k^{i-2}}{(Dk^{i-1}+\theta)^2}$$

2. Sample Application

The above procedure is applied to the data shown in Table XII.

Table XII

Failure Rate Data

Error No.	X_i	Error No.	X_i
1	9	14	9
2	12	15	4
3	11	16	1
4	4	17	3
5	7	18	3
6	2	19	6
7	5	20	1
8	8	21	11
9	5	22	33
10	7	23	7
11	1	24	91
12	6	25	2
13	1	26	1

The initial guess for the solution to the three equations are D_0 = .2112, k_0 = .95125, and θ_0 = .0576. The first two correspond to the solution obtained by solving the Eqs. 61 and 62 with θ=0. The value θ_0 is obtained by evaluating the test-end value of the failure rate, that is $\theta_0=D_0k_0^{26}$.

These three values produced the starting point for solving Equations 61, 62, and 63, by the Brown method. The resultant values are:

$\hat{D} = .2211$

$\hat{k} = .9468$

$\hat{\theta} = .00106$

The compare with the values D=.2112 and k = .95125 which were obtained with the Geometric De-Eutrophication Process.

VI. A PRIORI RELIABILITY

One of the possible areas of application of random test cases is in the estimation of an a priori reliability. This can be done in at least two ways. One of these has been described by Moranda (18) in which an assumed distribution of input data is used to estimate the average (weighted) number of errors resident in a program before testing is completed.

In this application the number of instructions (vis a vis segments) executed can counted directly. If the sample cases are drawn randomly according to the fitting operational probability law, then a simple calculation will give what may be called the "average operational error content".

This is done by applying the "Programmers Poisson Parameter" which is a universal constant of 1 error per 50 lines of code. If a run is made using the cases generated by operational-like data and the result is a sequence of numbers $N_1, N_2, \ldots, N$ representing the number of instructions exercised in each run, then an estimate of the average error content is the product of 1/50 and the average number of instructions exercised.

Thus, explicitly

$$M = 1/50 \quad \sum_{i=1}^{m} \frac{1}{\ } \; N_i$$

is the estimated average error content.

It might be argued that the total error content is not so much dependent onthe instruction counts as it is on the number of logical paths which can be formed. Without arguing this point, one way or another, it is merely noted that the method of estimating the total number of logical paths is the only one known which provides this figure, and hence could be used to make this estimate.

Once this figure is arrived at, there is a new way of providing an estimate of reliability. The number of paths found by test (and which presumably

have furnished debugging information for their correction, and which then can be considered error free) subtracted from the estimated total number produce a number which represents the number proportional to the error count (estimate).

It should be noted that the average operational error content derived above is not related in any simple way to the total software error content, since the former figure depends on the probability law for the operational input as well as on the structure of the program, while the best guess as to the total error content of the package is 1/50 times the total number of instructions - some of which may seldom, if ever, be exercised.

Once threads through a program can be established, a similar application can be made on them. The complications due to loops and repeated instruction are the major problems in this respect. These can best be resolved by forming mutually exclusive sets, but no clear cut best choice for this process has been found.

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Appendix I
PROGRAM TESTING SYSTEM

I. INTRODUCTION

The Program Testing System (PTS) was designed as an aid in software reliability studies of FORTRAN programs. The PTS system uses the International Mathematical and Statistical Library, IMSL, to generate the random data needed by the reliability study.

There are four basic modules in the PTS system.

(1) TEST CASE PREPROCESSOR
The Test Case Preprocessor generates a driver program for the subroutines under study. It also generates the data for test cases under user control.

(2) PET PREPROCESSOR
The preprocessor module of the PET system is used to analyze the FORTRAN source code under study in order to identify the program branch points. This information is put on an intermediate file and is used to define program segments. It also instruments the source code in order to gather execution statistics.

(3) TEST CASE LIBRARY
The Test Case Library write an intermediate file with execution statistics for each test case. It is transparent to the user.

(4) SEGMENT POSTPROCESSOR
The Segment Postprocessor using the file generated by the PET Preprocessor defines the program segments and prints the reports needed in the reliability study.

Complete descriptions of modules (1), (3) and (4) are given in sections II, III and IV. The preprocessor is described in the PET Manual issued by McDonnell Douglas Automation Company.

Illustration I-1 shows the flow of control through the PTS system.

The user inputs the test case options desired on cards to the test case preprocessor which outputs a FORTRAN driver program. The subject program is input to the preprocessor which outputs instrumented source code and a file with syntactic information to be used by the postprocessor. Then the driver program and instrumented source are compiled and executed. A file of execution statistics is generated at this time. Then the segment postprocessor is executed using the execution and syntactic statistic files and generating the reports needed for the reliability study.

II. TEST CASE PREPROCESSOR

The Test Case Preprocessor constructs, under user control, the random data for test cases needed to drive the subroutines being studied. In addition, it automatically generates a FORTRAN program to execute the subroutines. A restriction of the study is that all input must be through the calling sequence of the subroutine.

Four types of random distributions may be specified: Uniform, Triangular, Beta and Truncated Normal. The user must supply the range of values that the input parameter may assume. The range must be on the field of real numbers. A most likely value (mode) may be input; otherwise, it is assumed to be at the center of the range. The random numbers are then mapped into the range or into the log of the range according to a linear interpolation.

One of the above distribution options is specified for each individual variable or array. The same options are in effect for all the test cases.

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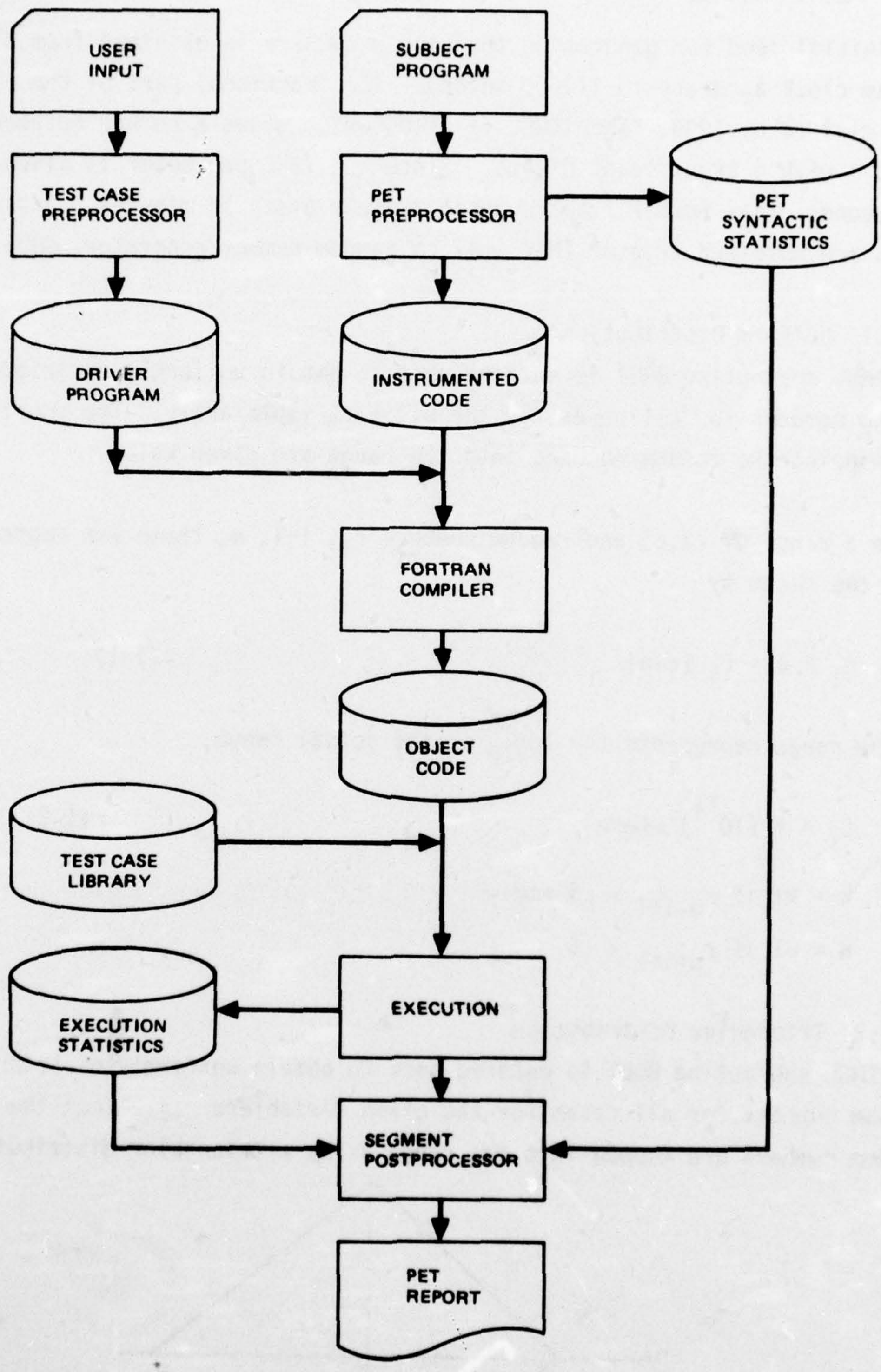


Figure I-1. Flow of Control

II.1 Random Methods Used

The initial seed for generating the random numbers is obtained from the system clock accurate to 1/1000 second. The fractional part of the time divided by 1000, TIME/1000, is used, which gives a number between 0 and 1 with 6 significant digits. Since the TIME parameter is given in seconds, this number repeats approximately every 15 minutes. Subsequent seeds are obtained fromthe IMSL uniform random number generator, GGU1.

II.1.1 Uniform Distribution

The IMSL subroutine GGU1 is entered once to obtain uniformly distributed random numbers for all cases for the given variable/array. The equations for mapping the random numbers into the range are given below.

Given a range of (a,c) and random numbers r_i, i=1, n, these are mapped into the range by

$$s_i = a + r_i (c-a) \tag{I-1}$$

If the range represents the $\log_{10}$ of the actual range,

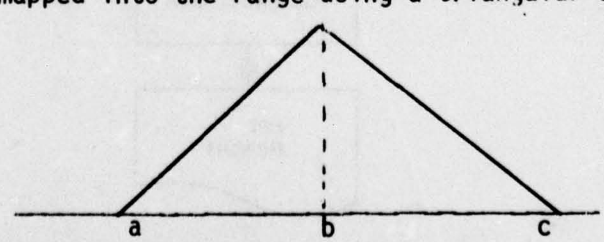
$$t_i = k\ (10^{r_i}) \text{ where} \tag{I-2}$$

$k = +1$ if $r_{n-i+1} > .5$ and

$k = -1$ if $r_{n=i+1} \leq .5$.

II.1.2 Triangular Distribution

The IMSL subroutine GGU1 is entered once to obtain uniformly distributed random numbers for all cases for the given variable/array. Then the random numbers are mapped into the range using a triangular distribution.



Given a range (a,c), a mode b, and an array of random numbers r_i, i=1, n, where $0 \leq r_i \leq 1$, the following equations are used.

If $o \leq r_i < \frac{b-a}{c-a}$, then

$$s_i = a + [r_i\ (c-a)\ (b-a)]^{1/2} \tag{I-3}$$

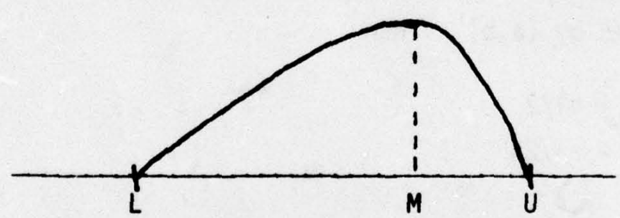
If $\frac{b-a}{c-a} \leq r_i \leq 1$, then

$$s_i = c - [(c-b)\ (c-a)\ (1-r_i)]^{1/2} \tag{I-4}$$

If the range represents the $\log_{10}$ of the number, then equation I-2 is used to obtain the actual numbers.

II.1.3 Beta Distribution

In order to obtain n β-distributed random numbers, the IMSL subroutine GGBET is entered n times. The GGBET routine requires a p and q which are multiples of 1/2. These are obtained using the following equations.



Let (L,U) be the range and M the mode.

Then $E\ (u) = \frac{1}{6}\ (L + 4M + U)$ (I-5)

$$m = (E\ (u) - L)/R \text{ where } R = U - L \tag{I-6}$$

$$P_s = m\ (36m\ (1-m) - 1) \tag{I-7}$$

and

$$q_s = (1-m)(36m\ (1-m) - 1) \tag{I-8}$$

Since p and q must be multiples of 1/2, the following approximation must be used:

$$p = \text{int}\ (2p_s + .5)/2 \qquad \text{(I-9)}$$

$$q = \text{int}\ (2q_s + .5)/2$$

where int = integer part.

If the range represents the $\log_{10}$ of the number, then

$t = k\ (10^r)$ where r is the β-distributed random number.

$k = \pm 1$ depending on the value of s(n).

where s(n) is a uniformly distributed random number obtained during the computation of 4.

II.1.4 Truncated Normal Distribution

The IMSL routine GGNOR is entered once to obtain normally distributed random numbers. Since a truncated normal is desired, the user must specify a range and the amount of truncation indicated by k. The mean will be assumed to be at the midpoint of the range. Let the range be given by (a,b). Then

$$m = (a + b)/2 \qquad \text{(I-10)}$$

$$\sigma = (b - a)/2k \qquad \text{(I-11)}$$

The normal variable r_i is mapped into the range using

$$s = \sigma . r_i + m \qquad \text{(I-12)}$$

For k = 1.96 about 1 in 20 numbers are not in the range and for k = 2.58, 1 in 100 are not in the range. To account for this, for $1.96 \leq k < 2.58$, 5% over the nominal amount of normal numbers are produced. And for k >2.58, 1% additional are produced.

If the s produced by I-12 is not in the range specified, it will be discarded and r_{i+1} will be tried.

III. TEST CASE LIBRARY

The Test Case Library writes a tape with execution statistics for each test case. It is loaded with the PET instrumented code and is transparent to the user in the PTS system. The library consists of three subroutines QERRQ, QPOST and QPCASE. Subroutine QPCASE writes case information on the tape every time it is called. Subroutine QPOST "wraps up" the run.

There are two records written on the tape for each case. The first record contains the branch and statement execution counts and the second record contains the variable assignment information.

IV. SEGMENT POSTPROCESSOR

The segment postprocessor defines the program segments and prints the reports necessary for the reliability study. The annotated source file fromthe PET preprocessor is used to define the FORTRAN program segments. There is a limit of 1000 segments which for a "typical" program would represent about 500 source statements. A complete description of a FORTRAN segment as defined in the PTS system is given in Section IV.2. There is no user input to the segment postprocessor. The reports generated by the segment postprocessor are described in detail in the following section.

IV.1 Postprocessor Reports

The first report contains the FORTRAN source listing followed by the statement numbers assigned by the PTS system. Each executable FORTRAN statement is assigned a number. A logical IF statement is assigned two numbers, one for the IF portion, and one for the true branch of the IF. This report allows the PTS user to correlate the program segments with the actual source statements.

The second report describes the FORTRAN segments as defined by the PTS system. Each segment consists of a set of statements that are executed sequentially. Two symbols are used in defining segments "," and "-". A "," indicates a branch and "-" indicates inclusive execution of the statements indicated. A "-1" is used to indicate an unresolved branch to a FORTRAN label. At the present time, it is left ot the PTS user to resolve such branches.

As an example, a segment described as (1-3,5) would include statements (1,2,3,5). The first statement of a segment is the last statement of its predecessor segment(s). Using the segment report, program paths may be constructed by the PTS user. The execution statistics for the segment are printed beside the segment description. At the end of the report, the percentage of the segments that were executed for each case are printed.

The third report shows the percentage of the cases that executed each segment. For instance, if segment 3 shows non-zero execution counts for 4 of 10 test cases, 40% would be printed.

The final report is a summary of the segments that were not executed. The percentage of the segments that were not executed for any test case is printed.

IV.2 Algorithm for Defining FORTRAN Program Segments

A program segment consists of a set of statements that are always executed sequentially and has one entrance and one exit. All segments have at least one predecessor segment except those starting with a subroutine entry. All segments have at least one successor segment except those ending on a return or halt.

There are four criteria for beginning a new segment.

(1) a subroutine or entry statement
(2) a statement label
(3) a DO statement
(4) termination of a previous segment

A segment will terminate if one or more of the following conditions are encountered:

(1) a RETURN or STOP statement
(2) a branching statement
(3) a CALL statement
(4) the end of a DO loop
(5) the beginning of a new segment

A FORTRAN multiple branch statement will cause several segments to be generated. For this reason, a given statement may be contained in more than one segment.

Illustration I-2 shows a sample of a FORTRAN program and the segments that were defined by the PTS system.

Segment 1 was terminated because a FORTRAN program need not return after a CALL statement. Segment 2 was terminated because of the statement label. Segments 3 and 4 represent the path taken by the FALSE and TRUE branch of the logical IF, respectively. Segment 9 terminated upon entry into the DO-loop, since the segment model being used differentiates between loops which "fall through" immediately and those which are iterated on. Segment 11 represents the path out of the DO-loop.

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	SAMPLE PROGRAM		SEGMENTS	
	STATEMENT	NO.	NO.	DESCRIPTION
	CALL OVERFL	1	1	1 – 2
	N=J	2	2	2 – 3
900	TAU = . ID-29	3	3	3 – 24, 26
	.	.	4	3 – 25, -1
	.	.	.	.
	.	.	.	.
	NN=N	23	.	.
	IF ( ) GO TO 6501	24 – 25	9	30 – 31
	.			
	.	26	10	31 – 32
	.	.		
		.	11	32 – 33
6560	DO 4000 I=1, M	30		
	DR(I) = AR(J)	31		
4000	DI(I) = Q(J)	32		
	RETURN	33		

Figure I-2. Sample FORTRAN Program and Segments

Appendix II

TABLES FOR THE DE-EUTROPHICATION PROCESS

Presented in the following set of tables are the estimates of model parameters and of their variances/covariances as well as estimates of the MTTF (at the end of the test) and the purification percentage.

These tables are derived from the data-derived ratio

$$R = \frac{\sum_{i=1}^{n} (i-1)X_i}{\sum_{i=1}^{n} X_i}$$

in the manner described in Section 5. Once R is calculated, values for all other parameters can be obtained by table lookup.

The technique as well as the use of the tables is described in Section 5, where sample table (Table XI on page 97) is explained. Footnotes on that table apply here.

N = 15

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
7.2	100.50	.1608	519.99	.8971	-465.9663	.0727
7.4	53.98	.3220	129.48	.8989	-115.8873	.0797
7.6	38.59	.4841	57.19	.9020	-51.0875	.0876
7.8	30.96	.6475	31.87	.9063	-28.3873	.0967
8.0	26.45	.8129	20.15	.9120	-17.8803	.1074
8.2	23.50	.9805	13.78	.9190	-12.1778	.1200
8.4	21.43	1.1510	9.95	.9277	-8.7458	.1351
8.6	19.92	1.3251	7.46	.9380	-6.5158	.1534
8.8	18.79	1.5034	5.79	.9500	-4.9898	.1761
9.0	17.90	1.6862	4.54	.9644	-3.9068	.2048
9.2	17.20	1.8754	3.64	.9807	-3.1019	.2426
9.4	16.64	2.0714	2.95	.9998	-2.4946	.2941
9.6	16.19	2.2758	2.42	1.0218	-2.0237	.3689
9.8	15.82	2.4897	2.00	1.0474	-1.6550	.4869
10.0	15.52	2.7151	1.66	1.0773	-1.3614	.7021
10.2	15.28	2.9545	1.39	1.1121	-1.1251	1.2221
10.4	15.07	3.2107	1.17	1.1528	-.9337	4.3337

Column 1 is the ratio $\Sigma(i-1)X_i \,/\, \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 16

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
8.0	50.39	.3774	97.26	.8709	-84.2061	.0770
8.2	38.41	.5297	49.27	.8740	-42.5688	.0643
8.4	31.82	.6831	29.54	.8763	-25.4517	.0925
8.6	27.69	.6383	19.54	.8837	-16.7772	.1021
8.8	24.87	.9957	13.79	.8902	-11.7885	.1132
9.0	22.85	1.1556	10.19	.8980	-8.6624	.1264
9.2	21.33	1.3186	7.78	.9072	-6.5738	.1422
9.4	20.17	1.4851	6.09	.9181	-5.1144	.1613
9.6	19.26	1.6559	4.86	.9305	-4.0527	.1851
9.8	18.54	1.6315	3.94	.9449	-3.2589	.2153
10.0	17.95	2.0130	3.23	.9613	-2.6488	.2550
10.2	17.47	2.2015	2.68	.9801	-2.1709	.3095
10.4	17.07	2.3976	2.23	1.0017	-1.7926	.3886
10.6	16.75	2.6037	1.87	1.0263	-1.4857	.5154
10.8	16.47	2.8201	1.58	1.0549	-1.2389	.7487
11.0	16.25	3.0493	1.34	1.0880	-1.0370	1.3273
11.2	16.06	3.2941	1.13	1.1264	-.8707	5.3138

Column 1 is the ratio $\Sigma(i-1)X_i \;/\; \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 17

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
8.2	128,16	,1417	712,14	,8420	-599,1572	,0635
8.4	68.32	,2837	177,54	,8434	-149,2391	,0687
8.6	48,47	,4264	78,48	,8455	-65,8611	,0745
8.8	38,63	,5698	43,85	,8488	-36,7176	,0811
9.0	32,79	,7146	27,80	,8528	-23,2121	,0886
9.2	28,95	,8609	19,08	,8578	-15,8789	,0972
9.4	26,25	1,0090	13,84	,8640	-11,4674	,1072
9.6	24.26	1,1595	10,43	,8712	-8,5956	,1188
9.8	22,75	1,3127	8,09	,8795	-6,6318	,1325
10.0	21,57	1,4691	6,42	,8892	-5,2273	,1469
10.2	20,63	1,6292	5,18	,9002	-4,1901	,1689
10.4	19,88	1,7934	4,24	,9128	-3,4044	,1937
10.6	19.26	1,9628	3,51	,9270	-2,7918	,2253
10.8	18,75	2,1377	2,93	,9433	-2,3101	,2669
11.0	18,33	2,3168	2,47	,9619	-1,9252	,3239
11.2	17,98	2,5079	2,09	,9829	-1,6110	,4075
11.4	17,68	2,7056	1,77	1,0069	-1,3546	,5409
11.6	17,43	2,9138	1,51	1,0343	-1,1420	,7903
11.8	17,22	3,1344	1,29	1,0656	-,9648	1,4268
12.0	17,05	3,3689	1,11	1,1018	-,8183	6,4287

Column 1 is the ratio $\Sigma(i-1)X_i \,/\, \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed-estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ø: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 10

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
9.0	62.73	.5350	130.94	.8202	-106.8948	.0667
9.2	47.51	.4699	66.45	.8225	-54.1557	.0721
9.4	39.12	.6057	39.89	.8255	-32.4321	.0782
9.6	33.84	.7426	26.47	.8294	-21.4577	.0850
9.8	30.23	.8810	18.74	.8342	-15.1387	.0928
10.0	27.63	1.0210	13.90	.8399	-11.1830	.1017
10.2	25.60	1.1632	10.66	.8465	-8.5333	.1120
10.4	24.16	1.3078	8.39	.8541	-6.6763	.1241
10.6	22.97	1.4554	6.73	.8627	-5.3242	.1383
10.8	22.01	1.6057	5.49	.8727	-4.3171	.1553
11.0	21.23	1.7601	4.53	.8838	-3.5387	.1761
11.2	20.58	1.9184	3.79	.8966	-2.9315	.2018
11.4	20.05	2.0817	3.19	.9108	-2.4455	.2347
11.6	19.60	2.2504	2.70	.9268	-2.0533	.2780
11.8	19.22	2.4257	2.30	.9449	-1.7312	.3378
12.0	18.90	2.6079	1.97	.9654	-1.4664	.4251
12.2	18.63	2.7986	1.69	.9885	-1.2458	.5656
12.4	18.40	2.9995	1.46	1.0147	-1.0598	.8314
12.6	18.21	3.2112	1.26	1.0446	-.9049	1.5158
12.8	18.04	3.4365	1.09	1.0787	-.7735	7.6774

Column 1 is the ratio $\Sigma(i-1)X_i \ / \ \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed-estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 19

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
9.2	159,15	,1267	941,55	,7961	-749,0902	,0563
9.4	84,32	,2536	234,85	,7971	-186,7059	,0604
9.6	59,47	,3810	103,93	,7987	-82,5185	,0649
9.8	47,13	,5089	58,14	,8011	-46,0807	,0698
10.0	39,79	,6377	36,94	,8042	-29,2121	,0754
10.2	34,95	,7677	25,42	,8079	-20,0442	,0817
10.4	31,54	,8989	18,48	,8124	-14,5182	,0887
10.6	29,02	1,0316	13,97	,8178	-10,9398	,0965
10.8	27,09	1,1665	10,88	,8238	-8,4747	,1060
11.0	25,58	1,3032	8,67	,8309	-6,7246	,1166
11.2	24,37	1,4428	7,03	,8387	-5,4204	,1291
11.4	23,39	1,5846	5,80	,8478	-4,4391	,1437
11.6	22,58	1,7301	4,83	,8578	-3,6709	,1614
11.8	21,91	1,8794	4,06	,8689	-3,0613	,1829
12.0	21,35	2,0324	3,44	,8816	-2,5746	,2095
12.2	20,88	2,1902	2,94	,8957	-2,1782	,2435
12.4	20,47	2,3533	2,52	,9116	-1,8510	,2883
12.6	20,13	2,5230	2,17	,9291	-1,5763	,3505
12.8	19,84	2,6795	1,87	,9488	-1,3473	,4418
13.0	19,59	2,8833	1,63	,9713	-1,1562	,5881
13.2	19,37	3,0771	1,41	,9963	-,9919	,8676
13.4	19,19	3,2811	1,23	1,0248	-,8536	1,5977
13.6	19,03	3,4979	1,07	1,0570	-,7348	8,9645

Column 1 is the ratio $\Sigma(i-1)X_i \,/\, \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 20

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
10.0	76.40	.3012	170,75	,7774	-132,2428	,0589
10.2	57.55	.4224	86,71	,7791	-67,0543	,0630
10.4	47.16	.5441	52,16	,7815	-40,2679	,0677
10.6	40.60	.6667	34,65	,7844	-26,6839	,0728
10.8	36.11	.7903	24,59	,7860	-18,8818	,0786
11.0	32.85	.9152	18,28	,7923	-13,9888	,0850
11.2	30.40	1.0415	14,06	,7972	-10,7167	,0923
11.4	28.50	1,1694	11,10	,8030	-8,4301	,1006
11.6	26.99	1.2996	8,94	,8092	-6,7545	,1101
11.8	25.77	1.4317	7,33	,8165	-5,5039	,1211
12.0	24.77	1.5664	6,08	,8246	-4,5425	,1339
12.2	23.94	1.7037	5,11	,8338	-3,7904	,1490
12.4	23.24	1.8444	4,33	,8439	-3,1867	,1672
12.6	22.66	1,9886	3,69	,8552	-2,6992	,1892
12.8	22.16	2,1369	3,17	,8677	-2,2960	,2167
13.0	21.73	2.2897	2,74	,8817	-1,9654	,2518
13.2	21.37	2.4485	2,37	,8969	-1,6836	,2985
13.4	21,06	2,6124	2,06	,9141	-1,4490	,3625
13.6	20.79	2.7827	1,80	,9335	-1,2508	,4564
13.8	20.55	2.9609	1,57	,9550	-1,0807	,6088
14.0	20,35	3,1462	1,37	,9790	-,9339	,9004
14.2	20.18	3.3448	1,20	1,0062	-,8098	1,6661
14.4	20.03	3,5536	1,05	1,0366	-,7017	10,0229

Column 1 is the ratio $\Sigma(i-1)X_i \;/\; \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 21

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
10.2	193.50	,1146	1210,65	,7571	-916,0569	,0506
10.4	101.98	,2293	301,98	,7578	-228,3303	,0539
10.6	71.59	,3443	133,79	,7591	-101,0581	,0574
10.8	56,47	,4598	74,93	,7610	-56,5231	,0613
11.0	47,46	,5759	47,66	,7633	-35,8842	,0656
11.2	41,51	,6929	32,84	,7660	-24,6640	,0704
11.4	37,30	,8108	23,93	,7696	-17,9195	,0757
11.6	34,19	,9299	18,13	,7736	-13,5333	,0816
11.8	31,79	1,0503	14,16	,7782	-10,5285	,0882
12.0	29,91	1,1723	11,31	,7834	-8,3771	,0957
12.2	28,41	1.2958	9,22	,7895	-6,7944	,1042
12.4	27,18	1,4213	7,62	,7963	-5,5913	,1139
12.6	26,16	1,5492	6,39	,8037	-4,6513	,1252
12.8	25,30	1,6800	5,39	,8118	-3,9033	,1384
13.0	24,58	1,8131	4,60	,8210	-3,3060	,1540
13.2	23,97	1,9494	3,95	,8312	-2,8175	,1726
13.4	23,45	2,0897	3,41	,8422	-2,4113	,1954
13.6	23,00	2,2332	2,96	,8548	-2,0763	,2235
13.8	22,62	2,3420	2,57	,8683	-1,7905	,2597
14.0	22,28	2,5353	2,25	,8834	-1,5498	,3074
14.2	21,99	2,6946	1,97	,9001	-1,3431	,3735
14.4	21,74	2,8600	1,73	,9187	-1,1670	,4707
14.6	21,52	3,0325	1,52	,9394	-1,0154	,6283
14.8	21,34	3,2134	1,34	,9626	-,8843	,9285
15.0	21,17	3,4033	1,18	,9885	-,7716	1,7236
15.2	21,03	3,6044	1,04	1,0174	-,6733	10,5589

Column 1 is the ratio $\Sigma(i-1)X_i \;/\; \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 22

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
11.0	91.40	.2736	216,98	,7408	-160,2254	,0527
11.2	68,55	.3836	110,26	,7420	-81,3201	,0560
11.4	55,94	,4939	66,41	,7440	-48,9103	,0597
11.6	47,97	,6050	44,19	,7463	-32,4798	,0637
11.8	42,49	,7168	31,40	,7490	-23,0251	,0681
12.0	38,52	,8294	23,39	,7524	-17,1010	,0730
12.2	35,52	,9433	18,02	,7561	-13,1322	,0784
12.4	33.19	1,0582	14,27	,7605	-10,3650	,0844
12.6	31,33	1,1748	11,53	,7653	-8,3372	,0913
12.8	29,82	1,2928	9,48	,7709	-6,8227	,0989
13.0	28,58	1,4123	7,90	,7772	-5,6620	,1077
13.2	27,54	1,5341	6,66	,7841	-4,7462	,1177
13,4	26,67	1,6584	5,67	,7915	-4,0120	,1292
13,6	25,92	1,7850	4,86	,7999	-3,4174	,1427
13,8	25,29	1,9145	4,19	,8090	-2,9280	,1587
14.0	24,75	2,0468	3,64	,8192	-2,5241	,1773
14,2	24,28	2,1828	3,17	,8302	-2,1826	,2010
14,4	23,87	2,3230	2,78	,8423	-1,8923	,2301
14,6	23,52	2,4667	2,44	,8559	-1,6485	,2669
14,8	23,21	2,6160	2,15	,8706	-1,4364	,3160
15.0	22,94	2,7703	1,90	,8869	-1,2547	,3835
15.2	22,71	2,9306	1,68	,9050	-1,0979	,4827
15,4	22,50	3,0982	1,48	,9249	-,9604	,6444
15,6	22,32	3,2734	1,31	,9470	-,8414	,9519
15,8	22,16	3,4574	1,17	,9717	-,7376	1,7728
16.0	22,02	3,6515	1,03	,9991	-,6471	11,0242

Column 1 is the ratio $\Sigma(i-1)X_i / \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 23

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
11.2	231.15	.1046	1520,33	,7232	-1098,9330	,0459
11.4	121.32	,2092	379,58	,7238	-274,2511	,0486
11.6	84,81	,3142	168,19	,7248	-121,4035	,0515
11.8	66,64	,4194	94,26	,7262	-67,9556	,0546
12.0	55,80	,5252	60,03	,7281	-43,2105	,0581
12.2	48,62	,6315	41,43	,7303	-29,7610	,0618
12.4	43,54	,7387	30,21	,7330	-21,6495	,0659
12.6	39,77	,8465	22,94	,7362	-16,3968	,0704
12,8	36,87	,9554	17,95	,7399	-12,7914	,0754
13.0	34,59	1,0655	14,39	,7440	-10,2158	,0810
13.2	32.75	1,1768	11,75	,7487	-8,3101	,0872
13.4	31.23	1,2896	9,74	,7538	-6,8585	,0942
13.6	29.98	1,4044	8,17	,7593	-5,7238	,1021
13.8	28,93	1,5205	6,94	,7658	-4,8344	,1110
14.0	28,04	1,6387	5,94	,7728	-4,1151	,1212
14.2	27,27	1,7595	5,12	,7803	-3,5235	,1330
14.4	26,62	1,8825	4,44	,7887	-3,0371	,1465
14.6	26,05	2,0082	3,87	,7979	-2,6314	,1631
14,8	25,56	2,1372	3,39	,8079	-2,2869	,1826
15.0	25,14	2,2693	2,99	,8189	-1,9952	,2064
15.2	24,76	2,4054	2,63	,8309	-1,7444	,2360
15,4	24,43	2.5461	2,33	,8438	-1,5263	,2740
15.6	24,15	2,6908	2,06	,8583	-1,3404	,3238
15,8	23.90	2,8405	1,83	,8743	-1,1791	,3925
16,0	23.68	2,9967	1.63	,8917	-1,0365	,4942
16.2	23,48	3,1590	1,45	,9110	-,9127	,6584
16.4	23,31	3,3287	1,29	,9323	-,8040	,9705
16.6	23,16	3,5068	1,15	,9558	-,7087	1,7976
16.8	23,02	3,6948	1,03	,9819	-,6242	10,8617

Column 1 is the ratio $\Sigma(i-1)X_i \,/\, \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 24

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
12.0	107.74	,2507	270,02	,7089	-190,9127	,0476
12.2	80,51	,3513	137,35	,7100	-97,0195	,0504
12.4	65,46	,4524	82,73	,7114	-58,3553	,0533
12.6	55,94	,5538	55,11	,7133	-38,8116	,0565
12.8	49,39	,6559	39,21	,7155	-27,5582	,0600
13.0	44,63	,7587	29,23	,7180	-20,4979	,0639
13.2	41,04	,8622	22,58	,7211	-15,7893	,0681
13.4	38,23	,9666	17,91	,7246	-12,4890	,0727
13.6	35,99	1,0721	14,51	,7285	-10,0823	,0778
13.8	34,16	1,1789	11,95	,7328	-8,2736	,0835
14.0	32,65	1,2872	9,98	,7376	-6,8806	,0899
14.2	31,39	1,3965	8,44	,7431	-5,7947	,0970
14.4	30,31	1,5082	7,20	,7487	-4,9146	,1050
14.6	29,41	1,6210	6,20	,7552	-4,2091	,1141
14.8	28,62	1,7360	5,37	,7623	-3,6273	,1246
15.0	27,95	1,8536	4,68	,7699	-3,1400	,1367
15.2	27,36	1,9734	4,10	,7782	-2,7325	,1507
15.4	26,85	2,0957	3,61	,7874	-2,3883	,1673
15.6	26,40	2,2214	3,19	,7973	-2,0913	,1873
15.8	26,01	2,3504	2,82	,8080	-1,8367	,2115
16.0	25,67	2,4831	2,51	,8197	-1,6158	,2418
16.2	25,36	2,6190	2,23	,8328	-1,4269	,2800
16.4	25,09	2,7604	1,99	,8467	-1,2581	,3311
16.6	24,86	2,9064	1,78	,8622	-1,1119	,4012
16.8	24,65	3,0577	1,59	,8791	-,9839	,5040
17.0	24,46	3,2156	1,42	,8977	-,8703	,6708
17.2	24,30	3,3805	1,27	,9181	-,7701	,9875
17.4	24,15	3,5532	1,14	,9406	-,6818	1,8211
17.6	24,03	3,7348	1,02	,9655	-,6038	10,2977

Column 1 is the ratio $\Sigma(i-1)X_i \,/\, \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed-estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N **and** ϕ**: in order to obtain the actual estimated covariance the entry should be divided by T.**

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 25

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
12.2	272.14	.0962	1873,68	,6935	-1298,8904	,0421
12.4	142,32	.1924	467,89	,6940	-324,2377	,0443
12.6	99,14	,2989	207,42	,6948	-143,6165	,0467
12,8	77,63	.3056	116,28	,6959	-80,4245	,0493
13.0	64,80	.4826	74,15	,6976	-51,2273	,0521
13,2	56,29	,5802	51,22	,6993	-35,3237	,0551
13,4	50,25	,6784	37,39	,7014	-25,7297	,0584
13,6	45,77	,7770	28,43	,7041	-19,5248	,0620
13,8	42,32	,8766	22,28	,7069	-15,2584	,0659
14.0	39,59	,9769	17,89	,7103	-12,2157	,0702
14.2	37,38	1,0785	14,63	,7139	-9,9536	,0749
14.4	35,57	1,1810	12,15	,7179	-8,2385	,0801
14.6	34,06	1,2846	10,23	,7225	-6,9084	,0859
14.8	32,79	1,3896	8,71	,7276	-5,8544	,0924
15.0	31.70	1.4966	7,47	,7328	-4,9946	,0997
15.2	30,78	1,6049	6,46	,7388	-4,2973	,1079
15.4	29,98	1.7148	5,63	,7454	-3,7237	,1171
15.6	29,28	1.8271	4,93	,7524	-3,2395	,1278
15.8	28,68	1,9412	4,34	,7602	-2,8339	,1400
16.0	28,15	2,0580	3,83	,7685	-2,4860	,1544
16.2	27,68	2,1775	3,40	,7775	-2,1871	,1713
16.4	27,27	2,3001	3,02	,7872	-1,9282	,1916
16,6	26,91	2,4258	2,69	,7978	-1,7041	,2163
16,8	26,59	2,5548	2,40	,8094	-1,5094	,2469
17,0	26,30	2,6877	2,15	,8221	-1,3388	,2859
17,2	26,05	2,8254	1,93	,8357	-1,1870	,3376
17,4	25,83	2,9672	1,73	,8508	-1,0553	,4083
17,6	25,63	3,1147	1,55	,8671	-,9375	,5126
17,8	25,45	3,2682	1,40	,8850	-,8331	,6809
18.0	25,29	3,4283	1,26	,9046	-,7406	,9982
18,2	25,15	3,5960	1,13	,9262	-,6583	1,8270
18.4	25,03	3,7717	1,01	,9499	-,5858	9.3922

Column 1 is the ratio $\Sigma(i-1)X_i \;/\; \Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed-estimate for step size: in order to determine the actual estimate of the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 26

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
13.0	125.40	.2313	330,06	,6808	-224,2022	,0435
13.2	93.42	.3241	167,99	,6817	-114,0230	,0458
13.4	75,72	,4172	101,28	,6829	-66,6705	,0482
13.6	64.51	.5107	67,49	,6844	-45,6908	,0508
13.8	56.80	.6046	48,07	,6862	-32,4853	,0537
14.0	51.19	.6991	35,88	,6883	-24,2005	,0568
14.2	46.94	,7942	27,74	,6907	-18,6666	,0601
14.4	43.62	,8899	22,04	,6936	-14,7968	,0638
14.6	40,95	,9866	17,87	,6966	-11,9618	,0678
14.8	38.78	1.0842	14,75	,7001	-9,8426	,0722
15.0	36,98	1,1826	12,36	,7041	-8,2155	,0770
15.2	35,47	1,2824	10,47	,7083	-6,9311	,0823
15,4	34,19	1,3836	8,95	,7129	-5,9020	,0882
15,6	33.10	1,4857	7,73	,7181	-5,0736	,0948
15,8	32,15	1,5898	6,72	,7235	-4,3850	,1022
16.0	31,34	1,6953	5,88	,7296	-3,8158	,1105
16.2	30,62	1.8027	5,17	,7361	-3,3350	,1200
16,4	30,00	1,9121	4,56	,7432	-2,9274	,1308
16,6	29,45	2.0236	4,05	,7508	-2,5795	,1433
16,8	28,96	2,1377	3,60	,7591	-2,2784	,1579
17.0	28,53	2,2541	3,21	,7681	-2,0185	,1750
17,2	28,15	2,3737	2,87	,7777	-1,7909	,1956
17,4	27,82	2,4959	2,58	,7883	-1,5935	,2205
17,6	27,52	2,6222	2,32	,7995	-1,4173	,2516
17,8	27,25	2,7519	2,08	,8119	-1,2630	,2912
18,0	27,01	2,8862	1,87	,8251	-1,1250	,3436
18,2	26,80	3,0247	1,69	,8396	-1,0035	,4154
18,4	26,61	3,1680	1,52	,8556	-,8964	,5200
18,6	26,44	3,3175	1,37	,8728	-,7996	,6895
18,8	26,29	3,4729	1,24	,8918	-,7142	1,0052
19,0	26,15	3,6360	1,12	,9123	-,6369	1,8243
19,2	26,03	3,8067	1,01	,9350	-,5685	8,7126

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 27

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
13.2	316.49	.0890	2272,37	,6672	-1515,7236	,0388
13.4	164.99	.1781	567,54	,6677	-378,4439	,0407
13.6	114.59	,2674	251,67	,6683	-167,6967	,0427
13.8	89.47	,3768	141,19	,6693	-93,9998	,0449
14.0	74.47	,4465	90,06	,6706	-59,8899	,0472
14.2	64.51	,5367	62,25	,6720	-41,3342	,0497
14.4	57.44	,6273	45,48	,6737	-30,1483	,0524
14.6	52.19	,7183	34,62	,6759	-22,9073	,0553
14.8	48.14	,8099	27,17	,6783	-17,9354	,0584
15.0	44.93	,9022	21,84	,6810	-14,3823	,0618
15.2	42.32	,9955	17,89	,6839	-11,7425	,0656
15.4	40.18	1,0896	14,83	,6871	-9,7380	,0696
15.6	38.39	1.1845	12,55	,6908	-8,1826	,0741
15.8	36.88	1.2806	10,70	,6948	-6,9481	,0790
16.0	35.60	1.3776	9,21	,6992	-5,9559	,0844
16.2	34.49	1.4761	7,98	,7040	-5,1415	,0904
16.4	33.54	1.5757	6,97	,7093	-4,4716	,0971
16.6	32.70	1.6773	6,12	,7148	-3,9028	,1046
16.8	31.97	1.7804	5,40	,7208	-3,4257	,1131
17.0	31.32	1.8951	4,79	,7274	-3,0199	,1227
17.2	30.76	1.9918	4,26	,7345	-2,6718	,1337
17.4	30.25	2.1009	3,81	,7420	-2,3682	,1464
17.6	29.80	2.2125	3,41	,7501	-2,1036	,1612
17.8	29.41	2.3261	3,06	,7591	-1,8757	,1786
18.0	29.05	2.4431	2,75	,7685	-1,6718	,1995
18.2	28.74	2.5628	2,48	,7788	-1,4933	,2249
18.4	28.45	2.6954	2,24	,7901	-1,3370	,2561
18.6	28.20	2,8121	2,02	,8021	-1,1966	,2960
18.8	27.97	2,9431	1,83	,8150	-1,0704	,3489
19.0	27.77	3,0780	1,65	,8292	-,9596	,4208
19.2	27.59	3,2184	1,50	,8444	-,8585	,5274
19.4	27.43	3,3637	1,36	,8611	-,7695	,6965
19.6	27.28	3,5148	1,23	,8794	-,6902	1,0096
19.8	27.15	3.6731	1,11	,8992	-,6181	1,6055
20.0	27.03	3.8386	1,01	,9209	-,5539	7,7113

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 28

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
14.0	144,39	,2147	397,42	,6558	-260,1252	,0400
14.2	107,28	,3006	202,39	,6566	-132,3920	,0419
14.4	86,71	,3372	122,04	,0575	-79,7458	,0440
14.6	73,70	,4738	81,42	,6588	-53,1448	,0462
14.8	64,73	,5608	58,02	,6603	-37,8104	,0486
15.0	58,19	,6483	43,34	,0620	-28,1974	,0511
15.2	53,23	,7362	33,55	,0641	-21,7839	,0538
15.4	49,35	,8247	26,67	,6663	-17,2794	,0568
15.6	46,24	,9138	21,67	,6690	-14,0057	,0000
15.8	43,70	1,0036	17,92	,6718	-11,5468	,0635
16.0	41,59	1,0943	15,02	,6750	-9,6522	,0673
16.2	39,81	1,1800	12,75	,6784	-8,1616	,0714
16.4	38,30	1,2786	10,92	,6821	-6,9691	,0759
16.6	37,01	1,3721	9,45	,6863	-6,0059	,0809
16.8	35,89	1,4671	8,23	,6907	-5,2068	,0864
17.0	34,91	1,5630	7,22	,6957	-4,5475	,0925
17.2	34,06	1,6606	6,36	,7009	-3,9882	,0993
17.4	33,31	1,7595	5,64	,7065	-3,5148	,1070
17.6	32,65	1,8604	5,01	,7125	-3,1070	,1156
17.8	32,07	1,9628	4,48	,7191	-2,7587	,1253
18.0	31,55	2,0670	4,01	,7262	-2,4562	,1364
18.2	31,08	2,1737	3,60	,7337	-2,1901	,1493
18.4	30,67	2,2824	3,24	,7419	-1,9561	,1642
18.6	30,30	2,3942	2,92	,7504	-1,7509	,1820
18.8	29,96	2,5081	2,64	,7598	-1,5697	,2030
19.0	29,67	2,6251	2,39	,7700	-1,4088	,2286
19.2	29,40	2,7452	2,17	,7809	-1,2656	,2603
19.4	29,16	2,8687	1,97	,7927	-1,1380	,3004
19.6	28,94	2,9965	1,79	,8053	-1,0225	,3534
19.8	28,75	3,1284	1,62	,8190	-,9192	,4261
20.0	28,58	3,2650	1,47	,8339	-,8266	,5318
20.2	28,42	3,4069	1,34	,8500	-,7428	,7010
20.4	28,28	3,5543	1,22	,8675	-,6678	1,0131
20.6	28,15	3,7080	1,11	,8866	-,6006	1,7831
20.8	28,04	3,8689	1,00	,9073	-,5399	6,9537

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 29

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTIF
14.2	364.17	.0829	2718,20	,6438	-1749,4325	,0360
14.4	189.31	.1658	678,65	,6441	-436,5859	,0376
14.6	131.14	,2488	301,16	,6447	-193,6438	,0393
14.8	102.14	,3320	169,03	,6455	-108,6126	,0412
15.0	84.80	,4155	107,83	,6465	-69,2094	,0431
15.2	73,29	,4993	74,58	,6477	-47,8043	,0452
15.4	65.11	,5834	54,54	,6492	-34,9106	,0475
15.6	59,02	,6678	41,55	,6509	-26,5475	,0499
15.8	54,32	,7528	32,63	,6529	-20,8124	,0525
16.0	50,59	,8384	26,25	,6551	-16,7064	,0552
16.2	47,57	,9245	21,54	,6576	-13,6708	,0583
16.4	45,08	1,0113	17,95	,6603	-11,3639	,0615
16.6	42,99	1,0990	15,15	,6633	-9,5624	,0651
16.8	41,22	1,1874	12,94	,6666	-8,1413	,0689
17.0	39,72	1,2767	11,16	,6703	-6,9937	,0731
17.2	38.41	1,3671	9,69	,6741	-6,0513	,0777
17.4	37,28	1,4587	8,47	,6782	-5,2688	,0828
17.6	36,30	1,5511	7,46	,6829	-4,6209	,0884
17.8	35.43	1,6454	6,60	,6876	-4,0652	,0946
18.0	34,66	1,7405	5,87	,6930	-3,5979	,1015
18,2	33,99	1,8371	5,24	,6988	-3,1960	,1092
18.4	33,38	1.9358	4,69	,7047	-2,8434	,1179
18,6	32,84	2,0361	4,21	,7112	-2,5382	,1278
18,8	32,36	2,1379	3,80	,7183	-2,2741	,1390
19,0	31,93	2,2423	3,43	,7257	-2,0378	,1521
19,2	31,55	2,3487	3,10	,7338	-1,8306	,1672
19.4	31,20	2,4576	2,81	,7424	-1,6465	,1849
19.6	30,89	2,5696	2,55	,7515	-1,4805	,2064
19,8	30,61	2,6838	2,32	,7615	-1,3345	,2321
20,0	30,35	2,8016	2,11	,7721	-1,2026	,2641
20.2	30.12	2,9222	1,92	,7837	-1,0855	,3045
20.4	29,92	3,0469	1,75	,7960	-,9792	,3576
20.6	29,73	3,1760	1,59	,8092	-,8828	,4307
20,8	29,56	3,3091	1,45	,8237	-,7969	,5360
21.0	29,41	3,4473	1,32	,8393	-,7189	,7034
21.2	29,28	3,5912	1,21	,8561	-,6479	1,0117
21.4	29,15	3,7406	1,10	,8745	-,5849	1,7511
21.6	29.04	3,6970	1,00	,8944	-,5275	6,1691

N = 30

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
15.0	164.72	.2004	472.50	.6334	-298.7872	.0370
15.2	122.09	.2807	240.70	.6341	-152.1264	.0387
15.4	98.46	.3612	145.22	.6349	-91.6995	.0404
15.6	83.49	.4419	96.92	.6360	-61.1410	.0423
15.8	73.17	.5230	69.11	.6372	-43.5380	.0443
16.0	65.64	.6044	51.66	.6386	-32.4925	.0464
16.2	59.92	.6861	40.02	.6404	-25.1303	.0487
16.4	55.44	.7684	31.84	.6422	-19.9555	.0512
16.6	51.84	.8512	25.89	.6444	-16.1929	.0538
16.8	48.90	.9346	21.43	.6468	-13.3670	.0566
17.0	46.45	1.0185	17.99	.6494	-11.1952	.0597
17.2	44.40	1.1031	15.30	.6524	-9.4922	.0630
17.4	42.64	1.1887	13.13	.6555	-8.1217	.0666
17.6	41.13	1.2750	11.37	.6589	-7.0113	.0705
17.8	39.82	1.3625	9.92	.6625	-6.0917	.0748
18.0	38.68	1.4505	8.72	.6666	-5.3363	.0794
18.2	37.68	1.5401	7.70	.6708	-4.6911	.0846
18.4	36.80	1.6304	6.84	.6755	-4.1501	.0902
18.6	36.02	1.7227	6.10	.6803	-3.6785	.0965
18.8	35.32	1.8161	5.46	.6855	-3.2745	.1035
19.0	34.70	1.9108	4.90	.6912	-2.9256	.1113
19.2	34.15	2.0070	4.42	.6973	-2.6213	.1201
19.4	33.65	2.1054	3.99	.7037	-2.3517	.1302
19.6	33.20	2.2055	3.61	.7106	-2.1145	.1416
19.8	32.80	2.3071	3.28	.7182	-1.9071	.1546
20.0	32.44	2.4114	2.98	.7261	-1.7199	.1699
20.2	32.11	2.5179	2.71	.7346	-1.5538	.1878
20.4	31.82	2.6274	2.47	.7436	-1.4029	.2093
20.6	31.55	2.7395	2.25	.7533	-1.2682	.2354
20.8	31.31	2.8546	2.05	.7637	-1.1469	.2676
21.0	31.09	2.9726	1.88	.7750	-1.0391	.3080
21.2	30.89	3.0945	1.72	.7870	-.9403	.3613
21.4	30.72	3.2206	1.57	.7999	-.8506	.4343
21.6	30.55	3.3507	1.43	.8139	-.7700	.5393
21.8	30.41	3.4856	1.31	.8289	-.6968	.7050
22.0	30.27	3.6259	1.20	.8451	-.6298	1.0075
22.2	30.15	3.7716	1.10	.8628	-.5701	1.7187
22.4	30.05	3.9237	1.00	.8819	-.5156	5.5675

N = 31

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
15.2	415.14	.0775	3211.43	.6226	-1998.8055	.0336
15.4	215.32	.1551	802.26	.6229	-499.2111	.0350
15.6	148.62	.2327	356.06	.6234	-221.4577	.0365
15.8	115.64	.3105	199.83	.6240	-124.1987	.0381
16.0	95.79	.3885	127.53	.6248	-79.1855	.0397
16.2	82.62	.4667	88.28	.6259	-54.7617	.0415
16.4	73.26	.5452	64.60	.6272	-40.0192	.0434
16.6	66.27	.6241	49.23	.6286	-30.4491	.0454
16.8	60.88	.7033	38.69	.6303	-23.8927	.0476
17.0	56.59	.7830	31.16	.6321	-19.2066	.0499
17.2	53.11	.8632	25.58	.6342	-15.7294	.0524
17.4	50.24	.9439	21.35	.6365	-13.0968	.0551
17.6	47.64	1.0253	18.05	.6390	-11.0419	.0579
17.8	45.80	1.1071	15.43	.6419	-9.4189	.0610
18.0	44.05	1.1899	13.32	.6448	-8.1028	.0644
18.2	42.54	1.2737	11.58	.6479	-7.0216	.0680
18.4	41.23	1.3579	10.16	.6515	-6.1363	.0720
18.6	40.08	1.4433	8.96	.6553	-5.3901	.0763
18.8	39.06	1.5298	7.94	.6593	-4.7578	.0811
19.0	38.17	1.6170	7.08	.6638	-4.2232	.0862
19.2	37.37	1.7060	6.32	.6682	-3.7561	.0920
19.4	36.66	1.7961	5.68	.6732	-3.3542	.0984
19.6	36.03	1.8871	5.12	.6786	-3.0007	.1054
19.8	35.46	1.9801	4.62	.6842	-2.7013	.1133
20.0	34.94	2.0748	4.18	.6901	-2.4298	.1223
20.2	34.48	2.1709	3.80	.6966	-2.1915	.1324
20.4	34.06	2.2689	3.45	.7035	-1.9801	.1439
20.6	33.69	2.3685	3.15	.7109	-1.7930	.1570
20.8	33.34	2.4712	2.87	.7185	-1.6207	.1726
21.0	33.04	2.5752	2.62	.7270	-1.4708	.1906
21.2	32.75	2.6822	2.40	.7359	-1.3339	.2121
21.4	32.50	2.7920	2.19	.7454	-1.2099	.2383
21.6	32.27	2.9044	2.01	.7557	-1.0986	.2704
21.8	32.06	3.0207	1.84	.7665	-.9958	.3116
22.0	31.87	3.1401	1.69	.7782	-.9037	.3651
22.2	31.70	3.2628	1.55	.7909	-.8213	.4372
22.4	31.54	3.3902	1.42	.8043	-.7449	.5422
22.6	31.40	3.5219	1.30	.8189	-.6761	.7062
22.8	31.27	3.6565	1.19	.8347	-.6135	.9997
23.0	31.16	3.8008	1.09	.8516	-.5561	1.6855
23.2	31.05	3.9490	1.00	.8699	-.5044	5.0504

N = 32

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
16.0	186,39	,1876	555,52	,6132	-340,1372	,0345
16.2	137,85	,2630	283,04	,6138	-173,2259	,0359
16.4	110,95	,3385	170,83	,6145	-104,4742	,0374
16.6	93,88	,4141	114,04	,6154	-69,6793	,0390
16.8	82,12	,4899	81,37	,6164	-49,6601	,0407
17.0	73,53	,5661	60,86	,6176	-37,0894	,0425
17.2	67,00	,6425	47,17	,6191	-28,7089	,0445
17.4	61,88	,7194	37,56	,6206	-22,8142	,0465
17.6	57,77	,7966	30,58	,6225	-18,5415	,0487
17.8	54,39	,8744	25,32	,6245	-15,3198	,0511
18.0	51,59	,9527	21,28	,6267	-12,8491	,0536
18,2	49,22	1,0315	18,11	,6292	-10,9049	,0563
18.4	47,20	1,1111	15,56	,6317	-9,3428	,0592
18.6	45,46	1,1911	13,50	,6347	-8,0845	,0623
18.8	43,96	1,2720	11,80	,6378	-7,0447	,0657
19.0	42,64	1,3538	10,38	,6411	-6,1751	,0694
19.2	41,47	1,4367	9,18	,6445	-5,4392	,0735
19.4	40,45	1,5202	8,17	,6484	-4,8203	,0779
19.6	39,54	1,6046	7,30	,6526	-4,2928	,0826
19.8	38,73	1,6906	6,55	,6568	-3,8302	,0879
20.0	38,01	1,7772	5,90	,6616	-3,4346	,0937
20.2	37,35	1,8655	5,33	,6665	-3,0846	,1001
20.4	36,77	1,9551	4,82	,6718	-2,7780	,1073
20.6	36,24	2,0460	4,38	,6774	-2,5082	,1153
20.8	35,76	2,1385	3,98	,6835	-2,2686	,1242
21.0	35,33	2,2331	3,63	,6897	-2,0530	,1345
21.2	34,94	2,3291	3,31	,6966	-1,8624	,1461
21.4	34,58	2,4272	3,03	,7038	-1,6904	,1594
21.6	34,26	2,5269	2,78	,7116	-1,5379	,1748
21.8	33,97	2,6295	2,54	,7198	-1,3978	,1931
22.0	33,70	2,7348	2,33	,7284	-1,2700	,2149
22.2	33,46	2,8422	2,14	,7377	-1,1561	,2412
22.4	33,24	2,9522	1,97	,7478	-1,0532	,2733
22.6	33,04	3,0660	1,81	,7584	-,9578	,3145
22.8	32,85	3,1827	1,66	,7698	-,8722	,3677
23.0	32,69	3,3029	1,53	,7821	-,7943	,4398
23.2	32,54	3,4272	1,40	,7953	-,7230	,5433
23.4	32,40	3,5558	1,29	,8094	-,6581	,7041
23.6	32,27	3,6896	1,19	,8245	-,5978	,9930
23.8	32,16	3,8281	1,09	,8409	-,5439	1,6411
24.0	32,06	3,9724	1,00	,8585	-,4948	4,5225

N = 33

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
16.2	469.51	.0728	3756,51	,6034	-2266,1870	,0315
16.4	242.97	.1456	938,11	,6036	-565,7329	,0327
16.6	167.59	,2166	416,50	,6040	-251,0845	,0340
16.8	129.98	,2916	233,86	,6046	-140,9014	,0354
17.0	107.47	,3648	149,31	,6053	-89,8832	,0368
17.2	92.51	,4382	103,38	,6062	-62,1731	,0383
17.4	81.88	.5118	75,69	,6073	-45,4670	,0400
17.6	73.94	,5857	57,70	,6085	-34,6156	,0417
17.8	67,81	,6599	45,40	,6100	-27,1976	,0435
18.0	62.93	,7345	36,58	,6115	-21,8718	,0455
18.2	58,96	,8096	30,05	,6133	-17,9360	,0476
18.4	55,69	,8849	25,10	,6153	-14,9532	,0498
18.6	52.94	,9609	21,24	,6174	-12,6255	,0522
18.8	50.61	1.0375	18,17	,6197	-10,7728	,0547
19.0	48.61	1.1145	15,71	,6223	-9,2871	,0575
19.2	46.88	1.1923	13,68	,6250	-8,0666	,0604
19.4	45,37	1,2706	12,01	,6280	-7,0600	,0636
19.6	44.04	1,3502	10,60	,6310	-6,2075	,0671
19.8	42,87	1,4302	9,41	,6344	-5,4924	,0708
20.0	41.84	1.5108	8,41	,6381	-4,8874	,0749
20.2	40,92	1.5930	7,53	,6419	-4,3582	,0793
20.4	40.09	1.6758	6,77	,6461	-3,9048	,0841
20.6	39.35	1,7599	6,11	,6504	-3,5073	,0895
20.8	38,69	1,8450	5,54	,6551	-3,1609	,0953
21.0	38,09	1,9314	5,03	,6601	-2,8550	,1018
21,2	37,54	2,0193	4,57	,6654	-2,5829	,1090
21.4	37,05	2,1086	4,17	,6710	-2,3419	,1171
21,6	36.60	2,1995	3,81	,6769	-2,1256	,1262
21.8	36.20	2.2922	3,48	,6832	-1,9312	,1365
22.0	35,83	2,3861	3,19	,6900	-1,7595	,1481
22,2	35.49	2,4828	2,93	,6970	-1,6006	,1617
22.4	35,19	2,5811	2,69	,7046	-1,4589	,1773
22,6	34,91	2,6813	2,47	,7127	-1,3314	,1955
22.8	34,65	2,7840	2,27	,7213	-1,2155	,2172
23.0	34,42	2,8897	2.09	,7304	-1,1082	,2437
23.2	34,21	2,9974	1,93	,7403	-1,0127	,2758
23,4	34.02	3.1065	1,78	,7507	-,9246	,3166
23,6	33,84	3.2230	1,64	,7618	-,8434	,3699
23.8	33,68	3.3408	1,51	,7737	-,7697	,4416
24.0	33,53	3,4625	1,39	,7864	-,7023	,5442
24,2	33,40	3,5862	1,28	,8001	-,6408	,7024
24.4	33,27	3,7185	1,18	,8148	-,5843	,9799
24.6	33,16	3,8542	1,09	,8305	-,5321	1,6002
24.8	33,06	3,9947	1,00	,8475	-,4854	4,1086

N = 34

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
17.0	209.40	.1767	646.73	.5948	-384.1749	.0323
17.2	154.55	.2475	329.44	.5952	-195.5955	.0335
17.4	124.15	.3185	198.88	.5958	-117.9960	.0348
17.6	104.86	.3895	132.84	.5966	-78.7595	.0362
17.8	91.58	.4608	94.84	.5976	-56.1789	.0377
18.0	81.87	.5324	70.98	.5986	-41.9919	.0392
18.2	74.47	.6042	55.02	.5998	-32.5033	.0409
18.4	68.67	.6764	43.84	.6012	-25.8586	.0426
18.6	64.00	.7485	35.70	.6027	-21.0245	.0445
18.8	60.18	.8217	29.60	.6044	-17.3952	.0465
19.0	56.99	.5950	24.90	.6063	-14.6045	.0486
19.2	54.30	.9687	21.21	.6085	-12.4147	.0509
19.4	52.00	1.0429	18.25	.6107	-10.6584	.0533
19.6	50.02	1.1178	15.85	.6131	-9.2291	.0559
19.8	48.29	1.1934	13.86	.6157	-8.0493	.0586
20.0	46.78	1.2696	12.21	.6184	-7.0671	.0616
20.2	45.45	1.3465	10.82	.6215	-6.2432	.0649
20.4	44.27	1.4242	9.64	.6246	-5.5400	.0684
20.6	43.23	1.5027	8.63	.6281	-4.9406	.0721
20.8	42.30	1.5816	7.76	.6320	-4.4279	.0762
21.0	41.45	1.6624	6.99	.6357	-3.9710	.0807
21.2	40.70	1.7435	6.33	.6399	-3.5800	.0856
21.4	40.02	1.8260	5.74	.6443	-3.2332	.0910
21.6	39.41	1.9095	5.23	.6490	-2.9280	.0969
21.8	38.85	1.9947	4.76	.6539	-2.6536	.1035
22.0	38.34	2.0809	4.35	.6591	-2.4112	.1107
22.2	37.88	2.1686	3.98	.6647	-2.1940	.1189
22.4	37.46	2.2577	3.65	.6706	-1.9993	.1280
22.6	37.08	2.3485	3.35	.6768	-1.8242	.1384
22.8	36.73	2.4410	3.08	.6835	-1.6658	.1501
23.0	36.41	2.5356	2.84	.6905	-1.5214	.1637
23.2	36.12	2.6318	2.61	.6980	-1.3915	.1793
23.4	35.85	2.7302	2.41	.7060	-1.2733	.1975
23.6	35.61	2.8314	2.22	.7143	-1.1641	.2196
23.8	35.39	2.9343	2.05	.7234	-1.0667	.2457
24.0	35.18	3.0406	1.89	.7329	-.9751	.2783
24.2	35.00	3.1496	1.75	.7430	-.8922	.3191
24.4	34.82	3.2614	1.62	.7539	-.8168	.3717
24.6	34.67	3.3766	1.49	.7656	-.7477	.4425
24.8	34.53	3.4961	1.38	.7778	-.6829	.5448
25.0	34.40	3.6188	1.27	.7912	-.6251	.6990
25.2	34.27	3.7466	1.18	.8053	-.5707	.9707
25.4	34.17	3.8784	1.09	.8206	-.5220	1.5463
25.6	34.07	4.0157	1.00	.8369	-.4768	3.7354

N = 35

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
17.2	527.15	,0686	4351,99	,5858	-2549,0763	,0296
17.4	272.33	,1373	1087,36	,5861	-636,7768	,0307
17.6	187,49	,2060	482,73	,5865	-282,6285	,0318
17.8	145.14	,2749	271,03	,5869	-158,5634	,0330
18.0	119.79	,3438	173,07	,5875	-101,1764	,0343
18.2	102.96	,4129	119,90	,5883	-70,0382	,0356
18.4	90.98	,4822	87,83	,5892	-51,2560	,0370
18.6	82.03	,5518	67,00	,5903	-39,0505	,0385
18.8	75.11	,6216	52,73	,5915	-30,6947	,0401
19.0	69.59	,6918	42,51	,5929	-24,7051	,0418
19.2	65.12	,7623	34,95	,5944	-20,2799	,0436
19.4	61.41	,8332	29,20	,5960	-16,9062	,0454
19.6	58.30	,9044	24,73	,5978	-14,2903	,0475
19.8	55.66	,9761	21,19	,5998	-12,2179	,0496
20.0	53,38	1,0484	18,32	,6019	-10,5382	,0519
20.2	51,42	1,1211	15,98	,6043	-9,1691	,0543
20.4	49,70	1,1945	14,04	,6068	-8,0323	,0569
20.6	48,20	1,2682	12,42	,6096	-7,0866	,0597
20.8	46,86	1,3428	11,04	,6125	-6,2819	,0628
21.0	45,68	1,4183	9,86	,6155	-5,5912	,0660
21.2	44,62	1,4946	8,85	,6187	-4,9976	,0696
21.4	43,67	1.5717	7,97	,6221	-4,4842	,0734
21.6	42,82	1,6492	7,21	,6260	-4,0433	,0775
21.8	42,05	1,7283	6,54	,6298	-3,6485	,0821
22.0	41,36	1,8083	5,94	,6339	-3,3014	,0870
22.2	40,73	1,8891	5,42	,6384	-2,9968	,0924
22.4	40,15	1,9713	4,95	,6430	-2,7239	,0984
22.6	39,64	2,0543	4,54	,6481	-2,4836	,1050
22.8	39,16	2,1389	4,15	,6533	-2,2654	,1123
23.0	38,73	2,2254	3,82	,6587	-2,0666	,1205
23.2	38,33	2,3126	3,52	,6647	-1,8914	,1297
23.4	37,97	2,4019	3,24	,6708	-1,7298	,1401
23.6	37,64	2,4932	2,99	,6772	-1,5826	,1520
23.8	37,34	2,5857	2,76	,6842	-1,4510	,1656
24.0	37,06	2,6805	2,54	,6916	-1,3296	,1813
24.2	36,81	2,7767	2,35	,6995	-1,2216	,1995
24.4	36,57	2,8759	2,18	,7077	-1,1197	,2215
24.6	36,36	2,9771	2,01	,7166	-1,0279	,2478
24.8	36,16	3,0813	1,86	,7259	-,9423	,2800
25.0	35,96	3,1884	1,72	,7357	-,8635	,3209
25.2	35,81	3,2979	1,60	,7464	-,7927	,3730
25.4	35,66	3,4113	1,48	,7575	-,7260	,4441
25.6	35,52	3,5276	1,37	,7696	-,6661	,5433
25.8	35,39	3,6481	1,27	,7825	-,6102	,6955
26.0	35,28	3,7727	1,17	,7962	-,5589	,9563
26.2	35,17	3,9020	1,08	,8109	-,5115	1,5090
26.4	35,07	4.0359	1.00	,8267	-,4684	3,4366

N = 36

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
18.0	233.74	.1669	746.38	.5780	-430.9003	.0303
18.2	172.22	.2337	380.25	.5784	-219.4196	.0314
18.4	138.12	.3007	229.60	.5789	-132.4085	.0326
18.6	116.48	.3678	153.37	.5795	-88.3817	.0338
18.8	101.56	.4350	109.55	.5804	-63.0830	.0351
19.0	90.64	.5025	82.00	.5813	-47.1684	.0364
19.2	82.34	.5702	63.63	.5823	-36.5555	.0378
19.4	75.81	.6382	50.71	.5835	-29.0917	.0394
19.6	70.56	.7064	41.33	.5848	-23.6762	.0410
19.8	66.26	.7749	34.29	.5864	-19.6115	.0427
20.0	62.66	.8438	28.86	.5880	-16.4784	.0444
20.2	59.62	.9132	24.59	.5897	-14.0123	.0464
20.4	57.02	.9831	21.17	.5916	-12.0364	.0484
20.6	54.76	1.0533	18.40	.5937	-10.4369	.0506
20.8	52.83	1.1241	16.12	.5959	-9.1187	.0529
21.0	51.11	1.1955	14.21	.5983	-8.0158	.0553
21.2	49.61	1.2672	12.61	.6010	-7.0975	.0580
21.4	48.27	1.3400	11.24	.6036	-6.3036	.0608
21.6	47.08	1.4129	10.08	.6066	-5.6363	.0639
21.8	46.01	1.4872	9.07	.6096	-5.0491	.0672
22.0	45.05	1.5619	8.19	.6130	-4.5441	.0708
22.2	44.19	1.6373	7.42	.6165	-4.1045	.0746
22.4	43.41	1.7137	6.75	.6203	-3.7166	.0788
22.6	42.70	1.7911	6.15	.6243	-3.3731	.0833
22.8	42.05	1.8698	5.62	.6283	-3.0651	.0883
23.0	41.47	1.9492	5.14	.6328	-2.7936	.0938
23.2	40.94	2.0296	4.72	.6375	-2.5516	.0998
23.4	40.45	2.1115	4.34	.6424	-2.3323	.1064
23.6	40.00	2.1950	3.99	.6475	-2.1329	.1139
23.8	39.59	2.2796	3.68	.6530	-1.9540	.1221
24.0	39.22	2.3655	3.40	.6588	-1.7926	.1313
24.2	38.88	2.4529	3.14	.6650	-1.6458	.1417
24.4	38.56	2.5424	2.90	.6714	-1.5106	.1537
24.6	38.27	2.6335	2.68	.6782	-1.3877	.1673
24.8	38.00	2.7267	2.48	.6854	-1.2748	.1831
25.0	37.76	2.8213	2.30	.6932	-1.1733	.2014
25.2	37.53	2.9186	2.14	.7013	-1.0786	.2232
25.4	37.33	3.0180	1.98	.7099	-.9920	.2495
25.6	37.14	3.1204	1.84	.7190	-.9113	.2818
25.8	36.96	3.2250	1.70	.7287	-.8382	.3220
26.0	36.80	3.3327	1.58	.7390	-.7702	.3741
26.2	36.65	3.4438	1.47	.7499	-.7072	.4442
26.4	36.52	3.5580	1.36	.7616	-.6495	.5426
26.6	36.39	3.6760	1.26	.7740	-.5962	.6917
26.8	36.28	3.7978	1.17	.7874	-.5473	.9436
27.0	36.17	3.9238	1.08	.8016	-.5026	1.4569
27.2	36.09	4.0550	1.00	.8168	-.4608	3.1616

N = 37

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
18.2	568.13	.0649	5000.87	.5697	-2848.6846	.0279
18.4	303.32	.1299	1249.57	.5699	-711.6779	.0289
18.6	206.47	.1949	554.70	.5702	-315.7976	.0299
18.8	161.14	.2599	311.64	.5707	-177.3514	.0310
19.0	132.60	.3251	199.04	.5712	-113.1988	.0321
19.2	113.95	.3905	137.89	.5719	-78.3571	.0333
19.4	100.55	.4560	101.05	.5727	-57.3752	.0345
19.6	90.52	.5217	77.10	.5736	-43.7232	.0358
19.8	82.77	.5876	60.70	.5746	-34.3842	.0372
20.0	76.60	.6537	48.98	.5759	-27.7092	.0386
20.2	71.57	.7202	40.30	.5772	-22.7636	.0402
20.4	67.41	.7871	33.68	.5786	-18.9931	.0418
20.6	63.91	.8542	28.54	.5801	-16.0651	.0435
20.8	60.94	.9217	24.47	.5819	-13.7450	.0453
21.0	58.39	.9897	21.18	.5837	-11.8710	.0472
21.2	56.17	1.0580	18.49	.5858	-10.3428	.0493
21.4	54.23	1.1270	16.25	.5878	-9.0666	.0515
21.6	52.53	1.1961	14.39	.5903	-8.0104	.0538
21.8	51.02	1.2661	12.81	.5928	-7.1102	.0563
22.0	49.68	1.3368	11.46	.5953	-6.3377	.0590
22.2	48.48	1.4081	10.29	.5981	-5.6749	.0619
22.4	47.40	1.4799	9.29	.6011	-5.1039	.0650
22.6	46.44	1.5523	8.41	.6043	-4.6075	.0683
22.8	45.56	1.6257	7.64	.6077	-4.1693	.0719
23.0	44.76	1.7002	6.96	.6111	-3.7798	.0758
23.2	44.04	1.7755	6.35	.6148	-3.4360	.0800
23.4	43.38	1.8515	5.81	.6188	-3.1327	.0846
23.6	42.78	1.9287	5.33	.6229	-2.8588	.0896
23.8	42.24	2.0068	4.90	.6273	-2.6150	.0951
24.0	41.74	2.0862	4.51	.6319	-2.3946	.1012
24.2	41.28	2.1663	4.16	.6369	-2.1981	.1079
24.4	40.86	2.2460	3.84	.6421	-2.0189	.1153
24.6	40.47	2.3313	3.55	.6475	-1.8541	.1236
24.8	40.11	2.4161	3.29	.6532	-1.7041	.1329
25.0	39.79	2.5019	3.05	.6593	-1.5696	.1433
25.2	39.49	2.5899	2.82	.6656	-1.4439	.1553
25.4	39.21	2.6790	2.62	.6725	-1.3312	.1688
25.6	38.95	2.7708	2.43	.6795	-1.2249	.1848
25.8	38.72	2.8641	2.26	.6870	-1.1289	.2032
26.0	38.50	2.9596	2.10	.6950	-1.0403	.2250
26.2	38.30	3.0571	1.95	.7035	-.9593	.2510
26.4	38.12	3.1576	1.81	.7123	-.8831	.2833
26.6	37.95	3.2605	1.68	.7218	-.8132	.3236
26.8	37.79	3.3659	1.56	.7319	-.7495	.3748
27.0	37.65	3.4748	1.45	.7429	-.6895	.4440
27.2	37.52	3.5867	1.35	.7539	-.6348	.5403
27.4	37.39	3.7022	1.26	.7660	-.5839	.6854
27.6	37.28	3.8214	1.17	.7789	-.5372	.9270
27.8	37.18	3.9446	1.08	.7927	-.4941	1.4120
28.0	37.08	4.0728	1.01	.8074	-.4540	2.0993

N = 38

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
19.0	259.41	.1581	854.73	.5625	-480.3135	.0286
19.2	190.83	.2214	435.41	.5628	-244.5556	.0296
19.4	152.83	.2848	263.00	.5633	-147.6421	.0306
19.6	128.70	.3483	175.76	.5639	-98.6137	.0317
19.8	112.04	.4120	125.53	.5646	-70.3724	.0328
20.0	99.87	.4758	94.03	.5654	-52.6668	.0340
20.2	90.59	.5399	72.94	.5662	-40.8057	.0352
20.4	83.31	.6041	58.19	.5673	-32.5164	.0365
20.6	77.43	.6686	47.43	.5684	-26.4659	.0379
20.8	72.62	.7333	39.37	.5698	-21.9405	.0394
21.0	68.59	.7985	33.16	.5712	-18.4448	.0409
21.2	65.19	.8639	28.27	.5727	-15.6997	.0426
21.4	62.27	.9297	24.37	.5744	-13.5035	.0443
21.6	59.76	.9958	21.19	.5762	-11.7224	.0461
21.8	57.57	1.0623	18.58	.5782	-10.2564	.0481
22.0	55.64	1.1296	16.39	.5802	-9.0243	.0502
22.2	53.94	1.1971	14.56	.5824	-7.9945	.0524
22.4	52.43	1.2654	12.99	.5847	-7.1141	.0548
22.6	51.09	1.3340	11.66	.5873	-6.3642	.0573
22.8	49.88	1.4033	10.50	.5899	-5.7163	.0600
23.0	48.79	1.4732	9.50	.5928	-5.1527	.0629
23.2	47.81	1.5440	8.62	.5957	-4.6565	.0660
23.4	46.93	1.6150	7.85	.5991	-4.2287	.0693
23.6	46.12	1.6874	7.16	.6023	-3.8421	.0730
23.8	45.39	1.7603	6.55	.6059	-3.5021	.0769
24.0	44.72	1.8341	6.01	.6097	-3.1994	.0811
24.2	44.11	1.9089	5.52	.6136	-2.9268	.0858
24.4	43.55	1.9848	5.09	.6178	-2.6811	.0909
24.6	43.03	2.0618	4.69	.6221	-2.4594	.0964
24.8	42.56	2.1394	4.33	.6268	-2.2622	.1025
25.0	42.13	2.2188	4.01	.6316	-2.0790	.1092
25.2	41.73	2.2992	3.71	.6367	-1.9141	.1167
25.4	41.36	2.3808	3.44	.6421	-1.7643	.1250
25.6	41.02	2.4640	3.19	.6478	-1.6268	.1343
25.8	40.71	2.5492	2.96	.6536	-1.4984	.1449
26.0	40.42	2.6354	2.75	.6600	-1.3831	.1569
26.2	40.15	2.7231	2.56	.6667	-1.2779	.1704
26.4	39.91	2.8130	2.38	.6737	-1.1796	.1862
26.6	39.68	2.9049	2.21	.6811	-1.0889	.2047
26.8	39.47	2.9986	2.06	.6889	-1.0058	.2264
27.0	39.28	3.0947	1.92	.6972	-.9284	.2526
27.2	39.10	3.1928	1.79	.7060	-.8578	.2843
27.4	38.94	3.2941	1.66	.7151	-.7910	.3244
27.6	38.78	3.3976	1.55	.7250	-.7303	.3752
27.8	38.64	3.5044	1.44	.7353	-.6732	.4434
28.0	38.51	3.6141	1.34	.7464	-.6208	.5380
28.2	38.40	3.7272	1.25	.7582	-.5725	.6785
28.4	38.29	3.8441	1.16	.7707	-.5273	.9119
28.6	38.18	3.9649	1.08	.7840	-.4859	1.3679
28.8	38.09	4.0902	1.01	.7981	-.4469	2.7029
29.0	38.01	4.2198	.94	.8134	-.4118	45.4114

N = 39

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
19.2	652,53	,0616	5706,73	,5550	-3166,5355	,0265
19.4	335,98	,1232	1425,50	,5551	-790,7590	,0273
19.6	230,59	,1848	633,03	,5554	-351,0616	,0282
19.8	177,97	,2466	355,56	,5557	-197,0890	,0292
20.0	146,47	,3084	227,22	,5562	-125,8853	,0302
20,2	125,52	,3703	157,48	,5568	-87,1936	,0312
20.4	110,59	,4324	115,38	,5575	-63,8246	,0323
20,6	99,45	,4946	88,12	,5584	-48,7040	,0334
20,8	90,80	,5571	69,36	,5592	-38,2874	,0346
21.0	83,93	,6196	55,98	,5602	-30,8676	,0359
21.2	78,33	,6827	46,08	,5614	-25,3721	,0372
21.4	73,69	,7458	38,55	,5627	-21,1962	,0386
21.6	69,79	,8093	32,68	,5640	-17,9391	,0401
21.8	66,46	,8732	28,03	,5655	-15,3560	,0417
22.0	63,61	,9373	24,28	,5671	-13,2758	,0434
22,2	61,13	1,0017	21,21	,5689	-11,5789	,0451
22,4	58,96	1,0667	18,67	,5708	-10,1661	,0470
22,6	57,05	1,1321	16,53	,5727	-8,9807	,0489
22,8	55,35	1,1980	14,72	,5748	-7,9789	,0510
23.0	53,85	1,2642	13,19	,5772	-7,1297	,0533
23.2	52,50	1,3312	11,86	,5796	-6,3929	,0557
23,4	51,29	1,3985	10,72	,5822	-5,7604	,0592
23,6	50,19	1,4666	9,71	,5850	-5,2045	,0609
23,8	49,20	1,5353	8,83	,5878	-4,7174	,0638
24.0	48,30	1,6051	8,05	,5907	-4,2826	,0670
24,2	47,48	1,6752	7,37	,5940	-3,9033	,0704
24,4	46,73	1,7462	6,75	,5973	-3,5631	,0740
24,6	46,05	1,8179	6,20	,6009	-3,2611	,0780
24,8	45,43	1,8905	5,71	,6047	-2,9898	,0823
25.0	44,86	1,9639	5,27	,6087	-2,7461	,0869
25,2	44,33	2,0387	4,87	,6127	-2,5230	,0920
25,4	43,84	2,1145	4,50	,6171	-2,3212	,0976
25,6	43,40	2,1909	4,17	,6218	-2,1411	,1037
25,8	42,99	2,2691	3,87	,6265	-1,9725	,1105
26,0	42,61	2,3478	3,59	,6317	-1,8229	,1180
26,2	42,26	2,4286	3,34	,6369	-1,6823	,1264
26,4	41,93	2,5105	3,10	,6425	-1,5545	,1357
26,6	41,64	2,5939	2,89	,6484	-1,4365	,1463
26,8	41,36	2,6788	2,69	,6546	-1,3288	,1583
27,0	41,10	2,7654	2,50	,6611	-1,2290	,1719
27,2	40,87	2,8533	2,34	,6681	-1,1385	,1876
27,4	40,65	2,9435	2,18	,6754	-1,0533	,2059
27,6	40,44	3,0362	2,03	,6829	-,9730	,2279
27,8	40,26	3,1302	1,89	,6911	-,9012	,2537
28,0	40,08	3,2272	1,77	,6996	-,8328	,2856
28,2	39,92	3,3263	1,65	,7087	-,7702	,3251
28,4	39,78	3,4279	1,54	,7183	-,7123	,3754
28,6	39,64	3,5325	1,43	,7284	-,6583	,4421
28,8	39,51	3,6404	1,34	,7391	-,6075	,5355
29,0	39,40	3,7513	1,25	,7505	-,5611	,6726
29,2	39,29	3,8657	1,16	,7627	-,5180	,8963

N = 39

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
29.4	39.19	3.9840	1.08	.7756	-.4781	1.3264
29.6	39.10	4.1063	1.01	.7893	-.4411	2.4953
29.8	39.01	4.2334	.94	.8039	-.4065	18.9020

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 40

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
20.0	286.39	.1502	971.58	.5482	-532.1015	.0270
20.2	210.41	.2103	495.19	.5485	-271.1069	.0279
20.4	168.27	.2705	299.14	.5489	-163.6966	.0288
20.6	141.51	.3308	199.89	.5494	-109.3232	.0298
20.8	123.04	.3912	142.81	.5500	-78.0470	.0308
21.0	109.52	.4519	106.96	.5507	-58.4008	.0318
21.2	99.25	.5125	83.09	.5516	-45.3322	.0329
21.4	91.15	.5735	66.27	.5524	-36.1149	.0341
21.6	84.62	.6347	54.04	.5534	-29.4127	.0353
21.8	79.27	.6960	44.89	.5546	-24.3996	.0366
22.0	74.79	.7577	37.81	.5558	-20.5197	.0379
22.2	71.00	.8197	32.26	.5571	-17.4793	.0394
22.4	67.76	.8819	27.82	.5586	-15.0498	.0409
22.6	64.96	.9444	24.22	.5602	-13.0761	.0424
22.8	62.50	1.0074	21.23	.5618	-11.4410	.0441
23.0	60.36	1.0708	18.76	.5637	-10.0842	.0459
23.2	58.46	1.1344	16.68	.5657	-8.9470	.0477
23.4	56.77	1.1986	14.90	.5677	-7.9744	.0497
23.6	55.26	1.2635	13.37	.5698	-7.1363	.0519
23.8	53.91	1.3284	12.07	.5723	-6.4236	.0541
24.0	52.69	1.3942	10.92	.5747	-5.7974	.0565
24.2	51.59	1.4606	9.92	.5773	-5.2497	.0591
24.4	50.58	1.5278	9.03	.5799	-4.7634	.0619
24.6	49.67	1.5954	8.26	.5829	-4.3394	.0648
24.8	48.84	1.6637	7.57	.5859	-3.9610	.0680
25.0	48.09	1.7325	6.95	.5892	-3.6270	.0714
25.2	47.39	1.8026	6.39	.5925	-3.3215	.0751
25.4	46.76	1.8731	5.90	.5961	-3.0516	.0790
25.6	46.17	1.9446	5.45	.5998	-2.8060	.0834
25.8	45.63	2.0169	5.04	.6038	-2.5852	.0880
26.0	45.13	2.0905	4.67	.6079	-2.3824	.0932
26.2	44.68	2.1651	4.33	.6122	-2.1982	.0988
26.4	44.26	2.2402	4.03	.6169	-2.0329	.1049
26.6	43.86	2.3169	3.74	.6217	-1.8798	.1117
26.8	43.50	2.3947	3.45	.6267	-1.7394	.1192
27.0	43.17	2.4743	3.24	.6319	-1.6086	.1276
27.2	42.86	2.5545	3.02	.6375	-1.4913	.1369
27.4	42.57	2.6366	2.82	.6433	-1.3809	.1475
27.6	42.30	2.7204	2.63	.6493	-1.2787	.1596
27.8	42.06	2.8054	2.45	.6558	-1.1859	.1732
28.0	41.83	2.8926	2.29	.6625	-1.0985	.1891
28.2	41.62	2.9809	2.14	.6698	-1.0193	.2073
28.4	41.42	3.0717	2.00	.6773	-.9446	.2289
28.6	41.24	3.1648	1.87	.6851	-.8745	.2550
28.8	41.07	3.2595	1.75	.6936	-.8111	.2862
29.0	40.92	3.3570	1.63	.7024	-.7512	.3254
29.2	40.77	3.4571	1.53	.7117	-.6951	.3755
29.4	40.64	3.5596	1.43	.7216	-.6439	.4409
29.6	40.51	3.6653	1.33	.7321	-.5958	.5315
29.8	40.40	3.7743	1.24	.7431	-.5506	.6058
30.0	40.29	3.8864	1.16	.7549	-.5093	.8801

N = 40

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
30.2	40.19	4.0024	1.08	.7674	-.4705	1.2873
30.4	40.10	4.1221	1.01	.7807	-.4348	2.3380
30.6	40.02	4.2462	.94	.7949	-.4016	11.6924

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 41

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
20.2	720.18	,0586	6467,08	,5412	-3499,6596	,0251
20.4	370.30	,1172	1615,53	,5413	-874,0199	,0259
20.6	253,83	,1758	717,65	,5416	-388,1920	,0267
20.8	195,64	,2345	403,13	,5419	-217,9623	,0276
21.0	160,81	,2933	257,64	,5424	-139,2361	,0285
21.2	137,62	,3522	178,54	,5428	-96,4231	,0294
21.4	121.11	,4112	130,84	,5434	-70,6041	,0304
21.6	108,78	,4703	99,94	,5442	-53,8881	,0314
21.8	99,21	,5296	78,73	,5450	-42,4077	,0324
22.0	91,59	,5891	63,56	,5459	-34,2008	,0335
22.2	85,39	,6489	52,33	,5469	-28,1240	,0347
22.4	80,24	,7088	43,79	,5480	-23,5030	,0360
22.6	75,92	,7690	37,16	,5492	-19,9147	,0372
22.8	72.24	,8294	31,90	,5506	-17,0678	,0386
23.0	69.06	,8901	27,65	,5520	-14,7666	,0400
23.2	66,30	,9513	24,16	,5535	-12,8790	,0416
23.4	63,89	1,0127	21,28	,5552	-11,3219	,0431
23.6	61,76	1,0746	18,85	,5569	-10,0109	,0448
23.8	59,86	1,1369	16,80	,5587	-8,9009	,0466
24.0	58,18	1,1995	15,06	,5607	-7,9593	,0485
24.2	56,67	1,2627	13,55	,5628	-7,1441	,0505
24.4	55,32	1,3260	12,26	,5651	-6,4463	,0527
24.6	54,09	1,3904	11,12	,5674	-5,8271	,0550
24.8	52,98	1,4551	10,12	,5699	-5,2883	,0574
25.0	51,97	1,5205	9,24	,5725	-4,8120	,0600
25.2	51,04	1,5864	8,46	,5752	-4,3903	,0628
25.4	50,20	1,6530	7,76	,5781	-4,0152	,0657
25.6	49,44	1,7197	7,15	,5814	-3,6853	,0689
25.8	48,74	1,7877	6,59	,5846	-3,3846	,0723
26.0	48,08	1,8565	6,08	,5879	-3,1120	,0760
26.2	47,49	1,9259	5,63	,5916	-2,8682	,0800
26.4	46,94	1,9963	5,22	,5953	-2,6460	,0844
26,6	46,43	2,0678	4,84	,5991	-2,4421	,0891
26,8	45,96	2,1402	4,49	,6032	-2,2572	,0943
27,0	45,52	2,2136	4,18	,6075	-2,0880	,0999
27,2	45,12	2,2878	3,89	,6121	-1,9349	,1061
27,4	44,75	2,3630	3,63	,6169	-1,7947	,1128
27,6	44,41	2,4396	3,39	,6219	-1,6642	,1204
27,8	44,09	2,5176	3,16	,6271	-1,5440	,1287
28,0	43,79	2,5974	2,95	,6325	-1,4309	,1382
28,2	43,51	2,6780	2,76	,6382	-1,3287	,1488
28,4	43,25	2,7601	2,58	,6443	-1,2343	,1607
28,6	43,01	2,8444	2,41	,6505	-1,1445	,1745
28,8	42,79	2,9298	2,25	,6572	-1,0631	,1902
29.0	42,59	3,0171	2,11	,6642	-,9870	,2086
29,2	42,40	3,1063	1,97	,6716	-,9165	,2301
29,4	42,22	3,1972	1,85	,6795	-,8518	,2556
29,6	42,06	3,2909	1,73	,6876	-,7898	,2871
29,8	41,91	3,3864	1,62	,6963	-,7333	,3255
30.0	41,77	3,4849	1,51	,7053	-,6796	,3750
30.2	41,63	3,5857	1,42	,7150	-,6302	,4396

N = 41

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
30.4	41.51	3.6894	1.33	.7252	-.5841	.5284
30.6	41.40	3.7959	1.24	.7360	-.5415	.6569
30.8	41.30	3.9060	1.16	.7474	-.5013	.8633
31.0	41.20	4.0198	1.08	.7595	-.4637	1.2465
31.2	41.11	4.1369	1.01	.7724	-.4294	2.1830
31.4	41.03	4.2583	.95	.7861	-.3972	8.3415

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 42

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
21.0	314.73	.1430	1098.00	.5350	-586.8743	.0256
21.2	230.94	.2002	559.64	.5352	-299.0231	.0264
21.4	184.46	.2576	338.10	.5356	-180.5720	.0273
21.6	154.95	.3150	226.01	.5360	-120.6495	.0281
21.8	134.54	.3725	161.43	.5365	-86.1067	.0290
22.0	119.63	.4302	120.96	.5371	-64.4732	.0299
22.2	108.27	.4880	93.93	.5378	-50.0213	.0309
22.4	99.34	.5458	75.01	.5387	-39.9092	.0319
22.6	92.14	.6040	61.19	.5396	-32.5191	.0330
22.8	86.21	.6624	50.82	.5406	-26.9732	.0341
23.0	81.26	.7209	42.83	.5416	-22.7054	.0353
23.2	77.06	.7798	36.56	.5428	-19.3532	.0366
23.4	73.47	.8388	31.55	.5441	-16.6771	.0379
23.6	70.37	.8981	27.48	.5456	-14.4999	.0393
23.8	67.65	.9578	24.11	.5470	-12.6987	.0407
24.0	65.26	1.0179	21.30	.5485	-11.1972	.0422
24.2	63.15	1.0783	18.94	.5502	-9.9347	.0438
24.4	61.27	1.1391	16.94	.5520	-8.8650	.0456
24.6	59.60	1.2000	15.23	.5541	-7.9552	.0473
24.8	58.08	1.2619	13.74	.5560	-7.1530	.0493
25.0	56.73	1.3237	12.45	.5583	-6.4707	.0513
25.2	55.50	1.3862	11.33	.5606	-5.8685	.0534
25.4	54.37	1.4497	10.32	.5628	-5.3292	.0558
25.6	53.36	1.5132	9.44	.5654	-4.8632	.0582
25.8	52.42	1.5776	8.66	.5680	-4.4438	.0608
26.0	51.57	1.6424	7.96	.5708	-4.0720	.0636
26.2	50.80	1.7076	7.34	.5738	-3.7420	.0666
26.4	50.08	1.7738	6.78	.5769	-3.4421	.0698
26.6	49.42	1.8408	6.27	.5801	-3.1708	.0733
26.8	48.81	1.9066	5.81	.5835	-2.9250	.0770
27.0	48.24	1.9772	5.39	.5870	-2.7013	.0810
27.2	47.72	2.0464	5.01	.5907	-2.5002	.0854
27.4	47.24	2.1167	4.66	.5946	-2.3145	.0901
27.6	46.80	2.1879	4.34	.5987	-2.1449	.0953
27.8	46.39	2.2598	4.05	.6031	-1.9916	.1009
28.0	46.00	2.3331	3.78	.6075	-1.8480	.1071
28.2	45.64	2.4079	3.52	.6121	-1.7144	.1140
28.4	45.31	2.4831	3.30	.6172	-1.5946	.1215
28.6	45.01	2.5599	3.08	.6223	-1.4822	.1299
28.8	44.72	2.6382	2.88	.6277	-1.3773	.1394
29.0	44.45	2.7179	2.70	.6333	-1.2803	.1500
29.2	44.21	2.7987	2.53	.6393	-1.1915	.1619
29.4	43.98	2.8814	2.37	.6455	-1.1078	.1756
29.6	43.76	2.9656	2.22	.6520	-1.0302	.1913
29.8	43.57	3.0512	2.08	.6590	-.9591	.2094
30.0	43.38	3.1389	1.95	.6663	-.8923	.2308
30.2	43.21	3.2289	1.83	.6738	-.8290	.2565
30.4	43.05	3.3206	1.71	.6819	-.7712	.2872
30.6	42.90	3.4147	1.61	.6903	-.7166	.3255
30.8	42.76	3.5113	1.51	.6992	-.6655	.3741
31.0	42.63	3.6104	1.41	.7086	-.6180	.4374

N = 42

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
31.2	42.51	3.7122	1.32	.7186	-.5736	.5241
31.4	42.40	3.8169	1.24	.7291	-.5324	.6489
31.6	42.30	3.9248	1.16	.7402	-.4938	.8459
31.8	42.21	4.0362	1.09	.7519	-.4576	1.2043
32.0	42.12	4.1513	1.02	.7644	-.4239	2.0540
32.2	42.04	4.2702	.95	.7776	-.3927	6.5840

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 43

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
21.2	791.17	,0558	7285,25	,5285	-3849,5027	,0239
21.4	406.34	,1117	1820,61	,5286	-961,8808	,0246
21.6	278.14	,1676	808,31	,5288	-426,9085	,0254
21.8	214.14	,2236	454,24	,5291	-239,8280	,0261
22.0	175.79	,2796	290,25	,5294	-153,1667	,0269
22.2	150.30	,3357	201,29	,5299	-106,1767	,0279
22.4	132.12	,3919	147,56	,5305	-77,7742	,0286
22.6	118.53	,4482	112,73	,5311	-59,3744	,0295
22.8	107.99	,5047	88,82	,5318	-46,7362	,0305
23.0	99.60	,5614	71,74	,5326	-37,7115	,0315
23.2	92.76	,6182	59,08	,5335	-31,0219	,0325
23.4	87.09	,6752	49,47	,5345	-25,9491	,0336
23.6	82.30	,7325	41,97	,5355	-21,9775	,0347
23.8	78.24	,7899	36,05	,5367	-18,8537	,0359
24.0	74,72	,8478	31.24	,5379	-16,3096	,0372
24.2	71,68	,9057	27,33	,5393	-14,2452	,0385
24.4	69.00	,9642	24.07	,5407	-12,5227	,0399
24.6	66,65	1.0227	21,36	,5423	-11,0920	,0414
24.8	64.55	1.0818	19.04	,5439	-9,8675	,0429
25.0	62.69	1.1409	17,09	,5458	-8,8393	,0445
25.2	61.01	1.2008	15,39	,5475	-7,9406	,0462
25.4	59.50	1.2611	13,92	,5495	-7,1630	,0481
25.6	58.13	1.3217	12,64	,5515	-6,4867	,0500
25.8	56.89	1.3829	11,51	,5537	-5,8928	,0520
26.0	55.77	1.4444	10,52	,5560	-5,3724	,0542
26.2	54.74	1,5065	9,64	,5584	-4,9077	,0565
26.4	53.81	1,5689	8,87	,5611	-4,4997	,0590
26.6	52.95	1,6321	8,17	,5638	-4,1313	,0616
26.8	52.15	1,6961	7,53	,5665	-3,7971	,0644
27.0	51.42	1,7607	6,97	,5694	-3,4979	,0674
27.2	50.75	1,8259	6,45	,5725	-3,2280	,0707
27.4	50.13	1,8917	5,99	,5758	-2,9840	,0741
27.6	49.55	1,9587	5,56	,5791	-2,7587	,0779
27.8	49.02	2.0261	5.17	,5827	-2,5564	,0819
28.0	48.53	2.0945	4,82	,5864	-2,3700	,0863
28.2	48,07	2,1637	4,50	,5902	-2,1999	,0911
28.4	47,65	2,2340	4,20	,5943	-2,0429	,0963
28.6	47,25	2,3051	3,92	,5985	-1,8992	,1020
28.8	46,69	2,3774	3,67	,6030	-1,7659	,1062
29.0	46,55	2,4507	3,43	,6076	-1,6431	,1151
29.2	46,23	2,5248	3,21	,6126	-1,5311	,1226
29.4	45,94	2,6003	3,01	,6177	-1,4269	,1309
29.6	45,66	2,6777	2,82	,6229	-1,3273	,1405
29.8	45,40	2,7559	2,65	,6286	-1,2368	,1510
30.0	45,17	2,8354	2,48	,6345	-1,1532	,1629
30.2	44,94	2,9171	2,33	,6405	-1,0733	,1766
30.4	44,73	2,9997	2,19	,6471	-1,0008	,1922
30.6	44,54	3,0845	2,05	,6538	-,9317	,2104
30.8	44,36	3,1706	1,93	,6610	-,8687	,2315
31.0	44,19	3,2590	1,81	,6684	-,8087	,2570
31.2	44.04	3,3494	1,70	,6762	-,7528	,2870

N = 43

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
31.4	43.89	3.4412	1,59	,6845	-,7007	,3254
31.6	43,76	3,5370	1,50	,6932	-,6515	,3734
31.8	43,63	3,6345	1,40	,7023	-,6057	,4359
32,0	43,52	3,7342	1,32	,7121	-,5635	,5199
32,2	43,41	3,6372	1,24	,7223	-,5235	,6417
32,4	43,31	3,9431	1,16	,7331	-,4862	,8312
32,6	43,21	4,0523	1,09	,7445	-,4513	1,1680
32,8	43,12	4,1648	1,02	,7566	-,4191	1,9260
33,0	43,04	4,2812	,96	,7694	-,3889	5,3134

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 44

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
22.0	344.42	.1365	1233.80	.5226	-644.3347	.0244
22.2	252.42	.1911	623.86	.5229	-328.3043	.0251
22.4	201.38	.2458	379.94	.5232	-198.2683	.0258
22.6	168.96	.3006	253.94	.5235	-132.4463	.0266
22.8	146.58	.3555	181.51	.5240	-94.6182	.0274
23.0	130.21	.4104	136.08	.5246	-70.8916	.0283
23.2	117.70	.4656	105.64	.5252	-54.9795	.0291
23.4	107.88	.5208	84.36	.5259	-43.8679	.0301
23.6	99.96	.5762	68.85	.5267	-35.7674	.0310
23.8	93.44	.6318	57.20	.5276	-29.6803	.0320
24.0	87.99	.6876	48.24	.5286	-25.0043	.0331
24.2	83.38	.7436	41.19	.5296	-21.3234	.0342
24.4	79.41	.7998	35.55	.5307	-18.3723	.0353
24.6	75.98	.8563	30.97	.5320	-15.9813	.0365
24.8	73.00	.9129	27.21	.5334	-14.0192	.0378
25.0	70.36	.9701	24.04	.5347	-12.3648	.0391
25.2	68.02	1.0274	21.39	.5362	-10.9821	.0405
25.4	65.95	1.0850	19.15	.5378	-9.8095	.0420
25.6	64.09	1.1431	17.22	.5395	-8.8015	.0435
25.8	62.42	1.2016	15.54	.5412	-7.9262	.0452
26.0	60.91	1.2603	14.10	.5432	-7.1739	.0469
26.2	59.55	1.3194	12.83	.5452	-6.5142	.0487
26.4	58.30	1.3792	11.71	.5473	-5.9285	.0507
26.6	57.18	1.4391	10.73	.5496	-5.4177	.0527
26.8	56.13	1.5000	9.84	.5518	-4.9545	.0549
27.0	55.19	1.5609	9.06	.5543	-4.5492	.0573
27.2	54.32	1.6225	8.36	.5570	-4.1844	.0597
27.4	53.51	1.6852	7.72	.5595	-3.8503	.0624
27.6	52.77	1.7482	7.15	.5623	-3.5519	.0652
27.8	52.09	1.8117	6.63	.5653	-3.2834	.0683
28.0	51.46	1.8757	6.17	.5685	-3.0413	.0715
28.2	50.87	1.9407	5.74	.5717	-2.8182	.0750
28.4	50.33	2.0064	5.35	.5751	-2.6146	.0787
28.6	49.82	2.0730	4.99	.5786	-2.4273	.0828
28.8	49.36	2.1403	4.66	.5823	-2.2566	.0872
29.0	48.92	2.2085	4.35	.5862	-2.0993	.0920
29.2	48.51	2.2781	4.07	.5901	-1.9520	.0972
29.4	48.14	2.3481	3.81	.5943	-1.8188	.1029
29.6	47.79	2.4190	3.57	.5988	-1.6962	.1091
29.8	47.46	2.4913	3.35	.6034	-1.5813	.1160
30.0	47.16	2.5648	3.14	.6082	-1.4742	.1236
30.2	46.87	2.6393	2.95	.6132	-1.3753	.1320
30.4	46.60	2.7154	2.77	.6184	-1.2822	.1414
30.6	46.36	2.7927	2.60	.6239	-1.1961	.1520
30.8	46.12	2.8713	2.44	.6297	-1.1163	.1640
31.0	45.91	2.9512	2.29	.6358	-1.0423	.1775
31.2	45.71	3.0325	2.16	.6422	-.9733	.1929
31.4	45.52	3.1158	2.03	.6489	-.9081	.2110
31.6	45.35	3.2011	1.91	.6558	-.8467	.2322
31.8	45.18	3.2884	1.79	.6630	-.7886	.2576
32.0	45.03	3.3772	1.68	.6707	-.7355	.2879

N = 44

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
32.2	44.89	3.4680	1.58	.6789	-.6862	.3249
32.4	44.75	3.5614	1.49	.6874	-.6390	.3721
32.6	44.63	3.6572	1.40	.6963	-.5949	.4333
32.8	44.52	3.7555	1.31	.7057	-.5539	.5157
33.0	44.41	3.8564	1.23	.7157	-.5155	.6332
33.2	44.31	3.9603	1.16	.7262	-.4795	.8141
33.4	44.22	4.0675	1.09	.7373	-.4457	1.1302
33.6	44.13	4.1779	1.02	.7490	-.4143	1.8189
33.8	44.05	4.2920	.96	.7614	-.3849	4.5069

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 45

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
22.2	865.50	.0534	8162,65	,5166	-4216,0646	,0228
22.4	443.98	.1067	2039,62	,5167	-1053,3010	,0235
22.6	303.60	.1601	905,95	,5169	-467,7587	,0241
22.8	233.46	.2136	508,96	,5171	-262,6788	,0248
23.0	191.46	.2671	325,34	,5174	-167,8418	,0256
23.2	163.51	.3207	225,58	,5178	-116,3168	,0263
23.4	143.60	.3744	165.47	,5184	-85,2799	,0271
23.6	128.70	.4281	126,43	,5190	-65,1146	,0279
23.8	117,15	,4821	99,64	,5196	-51,2748	,0288
24.0	107.94	.5361	80,49	,5203	-41,3805	,0296
24.2	100.42	.5904	66,30	,5210	-34,0481	,0306
24.4	94.19	,6448	55,53	,5219	-28,4833	,0315
24.6	88.95	,6993	47.15	,5229	-24,1622	,0325
24.8	84.47	,7542	40,48	,5239	-20,7136	,0336
25.0	80.61	,8092	35,12	,5250	-17,9424	,0347
25.2	77.26	,8644	30,73	,5262	-15,6792	,0359
25.4	74.32	,9199	27,08	,5275	-13,7956	,0371
25.6	71,72	,9756	24,04	,5289	-12,2255	,0384
25.8	69,42	1.0317	21.46	,5304	-10,8923	,0397
26.0	67.35	1.0882	19,25	,5319	-9,7492	,0411
26.2	65.51	1.1449	17,36	,5336	-8,7740	,0426
26.4	63.84	1.2020	15,71	,5353	-7,9227	,0442
26.6	62.33	1.2594	14,28	,5371	-7,1858	,0458
26.8	60,96	1,3174	13,02	,5390	-6,5332	,0476
27.0	59.71	1.3759	11,90	,5410	-5,9563	,0494
27.2	58.57	1.4343	10,92	,5433	-5,4556	,0514
27.4	57,53	1.4935	10,05	,5455	-5,0035	,0534
27.6	56.56	1.5537	9,25	,5477	-4,5921	,0557
27.8	55.68	1,6138	8,55	,5502	-4,2311	,0580
28.0	54.87	1.6748	7,91	,5527	-3,9015	,0605
28.2	54.12	1.7360	7,34	,5555	-3,6060	,0632
28.4	53.42	1.7983	6,81	,5583	-3,3369	,0660
28.6	52.78	1.8610	6,34	,5612	-3,0927	,0691
28.8	52.19	1.9241	5,91	,5644	-2,8719	,0723
29.0	51.63	1.9882	5,51	,5676	-2,6671	,0758
29.2	51.12	2.0532	5,14	,5709	-2,4789	,0796
29.4	50.64	2.1188	4,81	,5744	-2,3077	,0837
29.6	50.19	2.1851	4,50	,5781	-2,1500	,0881
29.8	49,78	2,2520	4,22	,5821	-2,0062	,0929
30.0	49,39	2,3206	3,95	,5860	-1,8695	,0981
30.2	49,03	2,3899	3,71	,5901	-1,7437	,1038
30.4	48,69	2,4598	3,48	,5946	-1,6291	,1100
30.6	48,38	2,5308	3,27	,5992	-1,5226	,1169
30.8	48,08	2,6035	3,07	,6039	-1,4208	,1245
31,0	47,81	2,6770	2,89	,6088	-1,3276	,1329
31,2	47,55	2,7520	2,71	,6140	-1,2398	,1424
31,4	47,31	2,8277	2,56	,6195	-1,1600	,1528
31.6	47,09	2,9051	2,40	,6252	-1,0844	,1647
31,8	46,88	2,9841	2,26	,6311	-1,0131	,1782
32,0	46,68	3,0646	2,13	,6374	-,9463	,1938
32.2	46,50	3,1466	2,00	,6439	-,8843	,2117

N = 45

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
32.4	46.33	3.2303	1,89	,6508	-,8264	,2326
32.6	46.17	3.3160	1,78	,6579	-,7715	,2576
32.8	46,02	3.4038	1,67	,6654	-,7195	,2879
33.0	45,86	3.4933	1,57	,6733	-,6717	,3247
33.2	45,75	3,5851	1,48	,6816	-,6266	,3710
33,4	45,63	3,6794	1,39	,6903	-,5841	,4512
33,6	45,52	3,7759	1,31	,6996	-,5446	,5115
33,8	45,41	3,8748	1,23	,7093	-,5079	,6243
34,0	45,31	3,9772	1,16	,7195	-,4726	,7995
34,2	45,22	4,0821	1,09	,7303	-,4404	1,0944
34,4	45,14	4,1905	1,02	,7417	-,4097	1,7205
34,6	45,06	4,3022	,96	,7537	-,3813	3,6910

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 46

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTIF
23.0	375.39	.1305	1378.70	.5111	-704.1230	.0233
23.2	274.85	.1826	702.98	.5113	-358.9503	.0239
23.4	219.05	.2351	424.73	.5116	-216.7855	.0246
23.6	183.61	.2875	283.95	.5119	-144.8668	.0253
23.8	159.12	.3399	203.01	.5124	-103.5208	.0260
24.0	141.20	.3925	152.17	.5129	-77.5431	.0268
24.2	127.54	.4451	118.24	.5135	-60.2136	.0276
24.4	116.79	.4979	94.41	.5141	-48.0373	.0284
24.6	108.11	.5508	77.07	.5148	-39.1790	.0292
24.8	100.97	.6039	64.05	.5156	-32.5234	.0301
25.0	94.99	.6572	54.02	.5164	-27.4004	.0311
25.2	89.93	.7107	46.14	.5173	-23.3751	.0320
25.4	85.59	.7642	39.86	.5184	-20.1673	.0330
25.6	81.83	.8181	34.73	.5195	-17.5503	.0341
25.8	78.54	.8722	30.51	.5206	-15.3903	.0352
26.0	75.64	.9266	26.98	.5218	-13.5889	.0364
26.2	73.08	.9812	24.02	.5232	-12.0796	.0376
26.4	70.80	1.0360	21.51	.5247	-10.7986	.0389
26.6	68.76	1.0911	19.36	.5262	-9.6985	.0403
26.8	66.92	1.1467	17.49	.5278	-8.7457	.0417
27.0	65.24	1.2028	15.86	.5293	-7.9086	.0432
27.2	63.74	1.2589	14.44	.5312	-7.1882	.0448
27.4	62.37	1.3155	13.20	.5331	-6.5535	.0464
27.6	61.11	1.3727	12.09	.5349	-5.9858	.0482
27.8	59.97	1.4301	11.11	.5370	-5.4861	.0501
28.0	58.92	1.4877	10.24	.5393	-5.0454	.0520
28.2	57.94	1.5466	9.44	.5414	-4.6370	.0541
28.4	57.06	1.6052	8.74	.5438	-4.2799	.0563
28.6	56.24	1.6645	8.11	.5464	-3.9569	.0587
28.8	55.47	1.7248	7.52	.5489	-3.6580	.0612
29.0	54.77	1.7851	7.00	.5515	-3.3924	.0639
29.2	54.12	1.8461	6.52	.5545	-3.1500	.0667
29.4	53.50	1.9083	6.08	.5573	-2.9236	.0698
29.6	52.94	1.9706	5.68	.5605	-2.7214	.0731
29.8	52.42	2.0340	5.31	.5636	-2.5322	.0766
30.0	51.93	2.0979	4.97	.5670	-2.3603	.0804
30.2	51.47	2.1625	4.65	.5706	-2.2022	.0845
30.4	51.05	2.2281	4.36	.5742	-2.0546	.0890
30.6	50.65	2.2947	4.09	.5779	-1.9179	.0938
30.8	50.28	2.3619	3.84	.5819	-1.7923	.0990
31.0	49.93	2.4296	3.62	.5862	-1.6781	.1046
31.2	49.61	2.4989	3.40	.5905	-1.5685	.1109
31.4	49.30	2.5692	3.20	.5950	-1.4673	.1178
31.6	49.02	2.6409	3.01	.5996	-1.3713	.1255
31.8	48.75	2.7132	2.83	.6046	-1.2842	.1338
32.0	48.51	2.7867	2.67	.6098	-1.2027	.1431
32.2	48.27	2.8622	2.51	.6150	-1.1238	.1538
32.4	48.06	2.9361	2.37	.6207	-1.0534	.1655
32.6	47.85	3.0161	2.23	.6265	-.9851	.1791
32.8	47.66	3.0954	2.10	.6327	-.9215	.1945
33.0	47.48	3.1760	1.98	.6391	-.8630	.2122

N = 46

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
33.2	47.32	3.2585	1.87	.6458	-.8071	.2331
33.4	47.16	3.3431	1.76	.6528	-.7540	.2580
33.6	47.01	3.4294	1.66	.6602	-.7047	.2877
33.8	46.88	3.5175	1.56	.6679	-.6586	.3240
34.0	46.75	3.6079	1.47	.6760	-.6150	.3697
34.2	46.63	3.7002	1.39	.6846	-.5748	.4278
34.4	46.52	3.7951	1.31	.6936	-.5366	.5059
34.6	46.42	3.8927	1.23	.7031	-.5005	.6161
34.8	46.32	3.9932	1.16	.7130	-.4665	.7833
35.0	46.23	4.0963	1.09	.7235	-.4348	1.0636
35.2	46.15	4.2026	1.03	.7345	-.4053	1.6338
35.4	46.07	4.3121	.97	.7463	-.3777	3.4238

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 47

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
23.2	943.21	.0511	9101.99	.5055	-4600.2636	.0218
23.4	483.36	.1022	2274.76	.5056	-1149.5703	.0224
23.6	330.12	.1533	1009.89	.5057	-510.1690	.0230
23.8	253.66	.2045	567.80	.5060	-286.7948	.0237
24.0	207.79	.2557	362.85	.5062	-183.1811	.0243
24.2	177.29	.3070	251.65	.5066	-126.9874	.0250
24.4	155.54	.3584	184.52	.5070	-93.0554	.0257
24.6	139.27	.4099	140.99	.5075	-71.0509	.0264
24.8	126.67	.4614	111.21	.5082	-56.0121	.0272
25.0	116.59	.5131	89.82	.5087	-45.1961	.0280
25.2	108.39	.5649	74.04	.5095	-37.2239	.0288
25.4	101.58	.6170	62.01	.5102	-31.1423	.0297
25.6	95.84	.6691	52.66	.5111	-26.4190	.0306
25.8	90.95	.7214	45.26	.5121	-22.6812	.0315
26.0	86.73	.7739	39.27	.5130	-19.6551	.0325
26.2	83.05	.8268	34.36	.5140	-17.1679	.0336
26.4	79.82	.8798	30.30	.5152	-15.1159	.0345
26.6	76.98	.9330	26.90	.5164	-13.3998	.0358
26.8	74.45	.9863	24.03	.5178	-11.9531	.0369
27.0	72.19	1.0400	21.58	.5192	-10.7134	.0382
27.2	70.15	1.0943	19.45	.5205	-9.6342	.0395
27.4	68.33	1.1484	17.62	.5222	-8.7167	.0408
27.6	66.66	1.2032	16.02	.5238	-7.9054	.0423
27.8	65.16	1.2581	14.62	.5255	-7.2017	.0438
28.0	63.78	1.3135	13.38	.5273	-6.5751	.0454
28.2	62.52	1.3695	12.28	.5292	-6.0168	.0471
28.4	61.37	1.4255	11.31	.5312	-5.5277	.0488
28.6	60.30	1.4824	10.42	.5332	-5.0800	.0507
28.8	59.33	1.5395	9.64	.5354	-4.6839	.0527
29.0	58.43	1.5968	8.93	.5378	-4.3308	.0548
29.2	57.60	1.6551	8.29	.5400	-4.0037	.0570
29.4	56.83	1.7138	7.71	.5425	-3.7100	.0594
29.6	56.11	1.7727	7.18	.5452	-3.4458	.0619
29.8	55.45	1.8325	6.69	.5479	-3.2013	.0646
30.0	54.83	1.8930	6.25	.5507	-2.9771	.0675
30.2	54.25	1.9539	5.84	.5536	-2.7735	.0705
30.4	53.72	2.0153	5.47	.5568	-2.5872	.0738
30.6	53.22	2.0776	5.13	.5600	-2.4145	.0774
30.8	52.75	2.1411	4.81	.5632	-2.2523	.0812
31.0	52.32	2.2050	4.51	.5667	-2.1044	.0853
31.2	51.91	2.2696	4.24	.5703	-1.9676	.0898
31.4	51.53	2.3349	3.99	.5742	-1.8421	.0946
31.6	51.17	2.4012	3.75	.5781	-1.7247	.0998
31.8	50.84	2.4683	3.53	.5823	-1.6154	.1055
32.0	50.53	2.5370	3.32	.5864	-1.5113	.1116
32.2	50.23	2.6063	3.13	.5909	-1.4159	.1187
32.4	49.96	2.6769	2.95	.5955	-1.3260	.1263
32.6	49.70	2.7479	2.78	.6005	-1.2445	.1346
32.8	49.46	2.8207	2.62	.6056	-1.1663	.1440
33.0	49.23	2.8950	2.47	.6107	-1.0917	.1546
33.2	49.02	2.9702	2.34	.6163	-1.0235	.1664

N = 47

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
33.4	48,83	3,0465	2,20	,6221	-,9601	,1796
33.6	48,64	3,1251	2,08	,6281	-,8983	,1952
33.8	48,47	3,2047	1,96	,6344	-,8417	,2129
34.0	48,30	3,2858	1,85	,6410	-,7888	,2334
34.2	48,15	3,3691	1,75	,6479	-,7380	,2560
34.4	48,01	3,4538	1,65	,6552	-,6912	,2871
34.6	47,87	3,5405	1,56	,6628	-,6469	,3228
34.8	47,75	3,6297	1,47	,6706	-,6044	,3660
35.0	47,63	3,7206	1,39	,6790	-,5655	,4249
35,2	47,52	3,8141	1,31	,6878	-,5283	,5015
35,4	47,42	3,9101	1,23	,6969	-,4932	,6086
35,6	47,33	4,0083	1,16	,7067	-,4610	,7662
35,8	47,24	4,1096	1,09	,7169	-,4303	1,0282
36.0	47,15	4,2139	1,03	,7277	-,4015	1,5463
36.2	47,08	4,3714	,97	,7390	-,3745	3,0440
36,4	47,00	4,4325	,91	,7510	-,3491	62,9168

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 48

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
24.0	407.74	.1251	1533.91	.5003	-766.9430	.0222
24.2	296.24	.1752	782.10	.5005	-390.9612	.0228
24.4	237.45	.2253	472.47	.5007	-236.0714	.0234
24.6	198.86	.2755	315.99	.5011	-157.8362	.0241
24.8	172.16	.3257	225.84	.5014	-112.7420	.0247
25.0	152.64	.3761	169.33	.5019	-84.4828	.0254
25.2	137.75	.4265	131.60	.5024	-65.6191	.0261
25.4	126.04	.4770	105.13	.5030	-52.3846	.0269
25.6	116.56	.5277	85.80	.5036	-42.7116	.0276
25.8	108.78	.5785	71.35	.5043	-35.4843	.0284
26.0	102.26	.6294	60.20	.5051	-29.9127	.0293
26.2	96.73	.6805	51.44	.5059	-25.5260	.0302
26.4	91.99	.7319	44.42	.5068	-22.0162	.0311
26.6	87.88	.7832	38.74	.5078	-19.1795	.0320
26.8	84.29	.8350	34.04	.5088	-16.8266	.0330
27.0	81.12	.8869	30.13	.5100	-14.8711	.0340
27.2	78.32	.9389	26.84	.5112	-13.2288	.0351
27.4	75.81	.9914	24.03	.5124	-11.8212	.0363
27.6	73.58	1.0440	21.63	.5138	-10.6248	.0375
27.8	71.56	1.0969	19.56	.5152	-9.5912	.0387
28.0	69.73	1.1502	17.75	.5167	-8.6871	.0400
28.2	68.08	1.2036	16.18	.5183	-7.9024	.0414
28.4	66.57	1.2576	14.79	.5200	-7.2056	.0428
28.6	65.19	1.3120	13.55	.5216	-6.5878	.0444
28.8	63.92	1.3667	12.46	.5234	-6.0397	.0460
29.0	62.77	1.4213	11.50	.5255	-5.5616	.0476
29.2	61.71	1.4767	10.62	.5275	-5.1256	.0494
29.4	60.72	1.5325	9.83	.5296	-4.7328	.0513
29.6	59.81	1.5891	9.12	.5317	-4.3749	.0533
29.8	58.96	1.6459	8.47	.5340	-4.0525	.0554
30.0	58.18	1.7033	7.89	.5364	-3.7598	.0577
30.2	57.46	1.7606	7.36	.5390	-3.4970	.0600
30.4	56.78	1.8195	6.86	.5414	-3.2503	.0626
30.6	56.15	1.8784	6.42	.5442	-3.0284	.0653
30.8	55.57	1.9380	6.01	.5470	-2.8235	.0682
31.0	55.02	1.9981	5.63	.5500	-2.6361	.0713
31.2	54.51	2.0590	5.28	.5530	-2.4628	.0746
31.4	54.04	2.1203	4.96	.5563	-2.3037	.0781
31.6	53.59	2.1825	4.66	.5596	-2.1555	.0819
31.8	53.18	2.2454	4.38	.5631	-2.0186	.0860
32.0	52.78	2.3094	4.12	.5666	-1.8896	.0905
32.2	52.42	2.3739	3.89	.5705	-1.7723	.0953
32.4	52.08	2.4396	3.66	.5743	-1.6599	.1006
32.6	51.75	2.5061	3.45	.5783	-1.5562	.1063
32.8	51.45	2.5732	3.25	.5827	-1.4613	.1125
33.0	51.17	2.6422	3.07	.5869	-1.3686	.1195
33.2	50.90	2.7114	2.90	.5916	-1.2852	.1270
33.4	50.66	2.7817	2.74	.5965	-1.2069	.1354
33.6	50.42	2.8533	2.58	.6015	-1.1331	.1447
33.8	50.20	2.9268	2.44	.6066	-1.0616	.1553
34.0	50.00	3.0009	2.30	.6120	-.9965	.1670

N = 48

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
34.2	49.80	3.0766	2.18	.6177	-.9348	.1804
34.4	49.62	3.1537	2.06	.6236	-.8767	.1957
34.6	49.45	3.2320	1.94	.6299	-.8230	.2131
34.8	49.29	3.3122	1.84	.6363	-.7716	.2337
35.0	49.14	3.3938	1.74	.6431	-.7237	.2577
35.2	49.00	3.4774	1.64	.6502	-.6782	.2866
35.4	48.87	3.5631	1.55	.6576	-.6350	.3220
35.6	48.75	3.6506	1.46	.6654	-.5946	.3660
35.8	48.63	3.7402	1.38	.6735	-.5566	.4220
36.0	48.53	3.8321	1.30	.6821	-.5207	.4962
36.2	48.42	3.9264	1.23	.6911	-.4868	.5995
36.4	48.33	4.0231	1.16	.7005	-.4554	.7508
36.6	48.24	4.1226	1.10	.7105	-.4256	.9975
36.8	48.15	4.2252	1.03	.7209	-.3973	1.4755
37.0	48.08	4.3308	.97	.7319	-.3710	2.7655
37.2	48.01	4.4395	.92	.7435	-.3465	18.6390

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 49

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
24.2	1024.12	.0490	10099.08	.4950	-4998.3878	.0209
24.4	524.31	.0980	2524.02	.4951	-1249.1008	.0215
24.6	357.81	.1471	1121.15	.4952	-554.7390	.0220
24.8	274.64	.1961	630.17	.4955	-311.7258	.0226
25.0	224.79	.2453	402.82	.4957	-199.1845	.0232
25.2	191.63	.2944	279.43	.4961	-138.1182	.0238
25.4	167.97	.3437	204.92	.4964	-101.2297	.0245
25.6	150.29	.3930	156.67	.4969	-77.3569	.0251
25.8	136.56	.4424	123.52	.4974	-60.9481	.0258
26.0	125.60	.4920	99.81	.4980	-49.2052	.0265
26.2	116.67	.5416	82.28	.4986	-40.5297	.0273
26.4	109.25	.5914	68.94	.4993	-33.9285	.0281
26.6	102.99	.6414	58.55	.5001	-28.7833	.0289
26.8	97.66	.6915	50.32	.5009	-24.7082	.0297
27.0	93.07	.7417	43.69	.5018	-21.4333	.0305
27.2	89.06	.7921	38.26	.5028	-18.7424	.0315
27.4	85.54	.8428	33.75	.5038	-16.5128	.0325
27.6	82.43	.8937	29.97	.5049	-14.6426	.0335
27.8	79.67	.9447	26.78	.5061	-13.0632	.0345
28.0	77.19	.9961	24.05	.5073	-11.7095	.0356
28.2	74.97	1.0478	21.69	.5086	-10.5452	.0368
28.4	72.96	1.0997	19.65	.5099	-9.5350	.0380
28.6	71.14	1.1519	17.88	.5114	-8.6568	.0392
28.8	69.49	1.2043	16.33	.5129	-7.8887	.0405
29.0	67.98	1.2571	14.95	.5146	-7.2102	.0419
29.2	66.60	1.3100	13.74	.5163	-6.6116	.0434
29.4	65.34	1.3636	12.65	.5181	-6.0736	.0449
29.6	64.16	1.4177	11.67	.5198	-5.5877	.0465
29.8	63.10	1.4715	10.81	.5219	-5.1638	.0482
30.0	62.11	1.5262	10.02	.5239	-4.7745	.0500
30.2	61.18	1.5815	9.31	.5260	-4.4208	.0519
30.4	60.33	1.6369	8.66	.5282	-4.1031	.0539
30.6	59.54	1.6930	8.07	.5305	-3.8113	.0560
30.8	58.81	1.7496	7.53	.5329	-3.5459	.0583
31.0	58.12	1.8069	7.04	.5353	-3.3011	.0607
31.2	57.48	1.8642	6.59	.5380	-3.0813	.0632
31.4	56.89	1.9226	6.17	.5407	-2.8751	.0660
31.6	56.33	1.9814	5.79	.5435	-2.6866	.0689
31.8	55.81	2.0408	5.44	.5464	-2.5124	.0720
32.0	55.33	2.1007	5.11	.5495	-2.3528	.0753
32.2	54.87	2.1614	4.81	.5527	-2.2043	.0788
32.4	54.44	2.2232	4.52	.5559	-2.0636	.0827
32.6	54.05	2.2849	4.27	.5595	-1.9382	.0867
32.8	53.67	2.3480	4.02	.5630	-1.8175	.0912
33.0	53.32	2.4119	3.79	.5667	-1.7054	.0961
33.2	52.99	2.4764	3.58	.5707	-1.6020	.1013
33.4	52.68	2.5421	3.38	.5747	-1.5041	.1070
33.6	52.38	2.6088	3.19	.5788	-1.4120	.1133
33.8	52.11	2.6765	3.01	.5831	-1.3258	.1202
34.0	51.85	2.7451	2.85	.5877	-1.2457	.1278
34.2	51.61	2.8143	2.69	.5925	-1.1718	.1361

N = 49

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
34.4	51.38	2,8851	2,55	,5974	-1,1010	,1454
34.6	51.17	2,9575	2,41	,6025	-1,0334	,1559
34.8	50.97	3,0304	2,28	,6079	-,9718	,1676
35.0	50.78	3.1053	2,15	,6134	-,9120	,1810
35.2	50.60	3,1809	2,04	,6193	-,8576	,1959
35.4	50,44	3,2563	1,93	,6254	-,8054	,2133
35.6	50.28	3.3375	1,82	,6317	-,7555	,2338
35,8	50.13	3,4182	1,72	,6384	-,7069	,2578
36.0	50,00	3.5006	1,63	,6453	-,6650	,2864
36.2	49.87	3.5848	1,54	,6526	-,6238	,3211
36.4	49,75	3,6708	1,46	,6602	-,5850	,3640
36.6	49,64	3,7590	1,38	,6682	-,5484	,4186
36,8	49,53	3,8494	1,30	,6766	-,5137	,4908
37,0	49,43	3.9423	1,23	,6853	-,4805	,5910
37,2	49,34	4,0374	1,16	,6945	-,4499	,7362
37.4	49,25	4,1351	1,10	,7042	-,4211	,9684
37.6	49.17	4,2357	1,04	,7144	-,3938	1,4034
37,8	49,09	4,3394	,98	,7251	-,3681	2,5058
38,0	49,02	4,4461	,92	,7364	-,3441	10,7436

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 50

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
25.0	441.38	.1201	1698.57	.4902	-832.0596	.0213
25.2	322.58	.1681	866.34	.4904	-424.3369	.0218
25.4	256.62	.2162	523.53	.4906	-256.3369	.0224
25.6	214.69	.2644	349.95	.4908	-171.2654	.0230
25.8	185.73	.3126	250.25	.4912	-122.4238	.0236
26.0	164.54	.3609	187.69	.4916	-91.7763	.0242
26.2	148.35	.4093	145.81	.4920	-71.2472	.0248
26.4	135.63	.4577	116.54	.4926	-56.9098	.0255
26.6	125.35	.5063	95.18	.4932	-46.4448	.0262
26.8	116.88	.5551	79.11	.4938	-38.5631	.0269
27.0	109.80	.6039	66.78	.4945	-32.5237	.0277
27.2	103.79	.6528	57.09	.4953	-27.7784	.0285
27.4	98.63	.7020	49.32	.4961	-23.9689	.0293
27.6	94.15	.7514	42.99	.4969	-20.8689	.0301
27.8	90.24	.8008	37.81	.4979	-18.3301	.0310
28.0	86.79	.8505	33.46	.4988	-16.1985	.0320
28.2	83.73	.9003	29.82	.5000	-14.4175	.0329
28.4	81.01	.9504	26.73	.5011	-12.9037	.0339
28.6	78.56	1.0008	24.06	.5023	-11.5931	.0350
28.8	76.36	1.0513	21.77	.5036	-10.4747	.0361
29.0	74.36	1.1023	19.76	.5049	-9.4888	.0372
29.2	72.55	1.1533	18.02	.5063	-8.6370	.0384
29.4	70.90	1.2047	16.48	.5079	-7.8860	.0397
29.6	69.39	1.2566	15.12	.5094	-7.2155	.0410
29.8	68.01	1.3085	13.91	.5110	-6.6264	.0424
30.0	66.74	1.3608	12.83	.5128	-6.0992	.0439
30.2	65.57	1.4136	11.86	.5146	-5.6247	.0454
30.4	64.48	1.4670	10.99	.5163	-5.1944	.0471
30.6	63.48	1.5206	10.20	.5183	-4.8089	.0488
30.8	62.55	1.5746	9.48	.5203	-4.4597	.0506
31.0	61.70	1.6286	8.84	.5225	-4.1470	.0525
31.2	60.90	1.6836	6.25	.5247	-3.8563	.0545
31.4	60.15	1.7389	7.71	.5270	-3.5924	.0566
31.6	59.46	1.7949	7.21	.5294	-3.3495	.0589
31.8	58.81	1.8511	6.76	.5319	-3.1281	.0613
32.0	58.21	1.9079	6.34	.5345	-2.9243	.0639
32.2	57.64	1.9655	5.95	.5372	-2.7348	.0666
32.4	57.11	2.0232	5.60	.5401	-2.5635	.0695
32.6	56.61	2.0822	5.26	.5429	-2.3996	.0726
32.8	56.15	2.1414	4.95	.5460	-2.2507	.0759
33.0	55.72	2.2011	4.67	.5493	-2.1134	.0795
33.2	55.31	2.2617	4.40	.5526	-1.9843	.0833
33.4	54.92	2.3232	4.16	.5560	-1.8637	.0874
33.6	54.56	2.3852	3.92	.5596	-1.7518	.0919
33.8	54.22	2.4485	3.70	.5632	-1.6452	.0968
34.0	53.90	2.5122	3.50	.5671	-1.5478	.1020
34.2	53.60	2.5774	3.31	.5709	-1.4535	.1078
34.4	53.32	2.6431	3.13	.5751	-1.3670	.1141
34.6	53.05	2.7093	2.96	.5795	-1.2875	.1208
34.8	52.80	2.7771	2.80	.5840	-1.2110	.1284
35.0	52.57	2.8461	2.65	.5886	-1.1384	.1368

N = 50

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
35.2	52,35	2.9160	2,51	,5934	-1,0708	,1461
35.4	52.14	2.9870	2,38	,5985	-1,0074	,1565
35.6	51,94	3,0592	2,25	,6038	-,9477	,1682
35.8	51,76	3,1327	2,13	,6093	-,8916	,1813
36.0	51,59	3,2075	2,02	,6151	-,8386	,1963
36.2	51,43	3,2840	1,91	,6210	-,7880	,2137
36.4	51.27	3,3619	1,81	,6273	-,7407	,2337
36.6	51.13	3,4415	1,71	,6338	-,6955	,2575
36.8	50,99	3,5228	1,62	,6406	-,6531	,2858
37.0	50,87	3,6056	1,54	,6477	-,6136	,3197
37.2	50,75	3,6907	1,45	,6551	-,5754	,3624
37.4	50,64	3,7775	1,38	,6629	-,5398	,4160
37.6	50,53	3,8662	1,30	,6711	-,5067	,4857
37.8	50.43	3,9575	1,23	,6797	-,4748	,5821
38.0	50,34	4,0510	1,16	,6887	-,4451	,7206
38.2	50.26	4,1475	1,10	,6981	-,4164	,9434
38.4	50,18	4,2461	1,04	,7080	-,3901	1,3421
38.6	50,10	4,3478	,98	,7184	-,3651	2,3013
38.8	50,03	4,4528	,93	,7293	-,3414	7,7463

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 51

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
25.2	1108.55	.0471	11165.20	.4852	-5417.0596	.0201
25.4	567.03	.0942	2790.53	.4853	-1353.7687	.0206
25.6	386.61	.1413	1239.46	.4854	-601.1754	.0211
25.8	296.46	.1884	696.47	.4856	-337.6920	.0216
26.0	242.46	.2356	445.32	.4858	-215.8520	.0222
26.2	206.50	.2829	308.83	.4861	-149.6256	.0227
26.4	180.88	.3301	226.60	.4865	-109.7397	.0233
26.6	161.70	.3775	173.24	.4869	-83.8589	.0239
26.8	146.81	.4250	136.61	.4874	-66.0827	.0246
27.0	134.93	.4725	110.42	.4879	-53.3745	.0252
27.2	125.25	.5202	91.05	.4885	-43.9771	.0259
27.4	117.20	.5679	76.30	.4891	-36.8231	.0266
27.6	110.41	.6159	64.83	.4898	-31.2573	.0273
27.8	104.62	.6639	55.74	.4906	-26.8470	.0281
28.0	99.62	.7121	48.38	.4913	-23.2769	.0289
28.2	95.26	.7605	42.37	.4922	-20.3581	.0297
28.4	91.44	.8091	37.39	.4931	-17.9443	.0306
28.6	88.06	.8577	33.23	.4942	-15.9284	.0315
28.8	85.05	.9067	29.69	.4952	-14.2103	.0324
29.0	82.36	.9558	26.68	.4963	-12.7507	.0334
29.2	79.93	1.0053	24.07	.4974	-11.4849	.0344
29.4	77.75	1.0548	21.84	.4987	-10.4016	.0354
29.6	75.77	1.1046	19.88	.5001	-9.4527	.0365
29.8	73.97	1.1547	18.15	.5014	-8.6167	.0377
30.0	72.31	1.2054	16.62	.5028	-7.8726	.0389
30.2	70.80	1.2560	15.28	.5043	-7.2213	.0402
30.4	69.42	1.3070	14.08	.5059	-6.6423	.0415
30.6	68.14	1.3586	13.00	.5075	-6.1162	.0429
30.8	66.97	1.4101	12.04	.5093	-5.6537	.0444
31.0	65.89	1.4619	11.18	.5112	-5.2357	.0460
31.2	64.88	1.5144	10.39	.5131	-4.8540	.0476
31.4	63.94	1.5672	9.68	.5151	-4.5090	.0493
31.6	63.07	1.6205	9.02	.5171	-4.1925	.0511
31.8	62.26	1.6742	8.43	.5192	-3.9050	.0530
32.0	61.51	1.7285	7.88	.5214	-3.6405	.0551
32.2	60.80	1.7831	7.38	.5237	-3.3996	.0572
32.4	60.14	1.8383	6.92	.5261	-3.1763	.0595
32.6	59.53	1.8936	6.50	.5287	-2.9751	.0619
32.8	58.95	1.9500	6.11	.5312	-2.7844	.0645
33.0	58.42	2.0065	5.75	.5340	-2.6122	.0672
33.2	57.91	2.0642	5.41	.5367	-2.4476	.0701
33.4	57.43	2.1220	5.10	.5396	-2.2963	.0732
33.6	56.99	2.1808	4.81	.5426	-2.1571	.0766
33.8	56.57	2.2399	4.54	.5458	-2.0280	.0802
34.0	56.18	2.2997	4.29	.5491	-1.9074	.0840
34.2	55.81	2.3601	4.05	.5526	-1.7956	.0881
34.4	55.46	2.4216	3.83	.5561	-1.6893	.0926
34.6	55.13	2.4841	3.62	.5597	-1.5888	.0975
34.8	54.83	2.5467	3.43	.5636	-1.4976	.1026
35.0	54.53	2.6110	3.25	.5675	-1.4090	.1084
35.2	54.26	2.6759	3.07	.5715	-1.3267	.1147

N = 51

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
35.4	54.00	2,7416	2,91	,5758	-1,2499	,1215
35.6	53,76	2,8084	2,76	,5803	-1,1772	,1290
35.8	53,53	2,8764	2,61	,5849	-1,1086	,1374
36.0	53,31	2,9455	2,48	,5896	-1,0435	,1467
36.2	53,11	3,0153	2,35	,5947	-,9837	,1569
36.4	52,92	3,0871	2,23	,5998	-,9252	,1687
36.6	52,74	3,1595	2,11	,6052	-,8713	,1817
36.8	52,57	3,2336	2,00	,6108	-,8199	,1967
37.0	52,41	3,3085	1,90	,6168	-,7726	,2136
37.2	52,26	3,3854	1,80	,6229	-,7267	,2336
37.4	52,12	3,4638	1,70	,6293	-,6835	,2569
37.6	51,99	3,5438	1,62	,6360	-,6427	,2846
37.8	51,87	3,6257	1,53	,6429	-,6037	,3184
38.0	51,75	3,7095	1,45	,6502	-,5669	,3601
38.2	51,64	3,7947	1,37	,6579	-,5329	,4120
38.4	51,54	3,8824	1,30	,6659	-,5001	,4804
38.6	51,44	3,9723	1,23	,6742	-,4691	,5736
38.8	51,35	4,0644	1,17	,6829	-,4399	,7073
39.0	51,26	4,1588	1,10	,6922	-,4127	,9142
39.2	51,18	4,2561	1,04	,7018	-,3866	1,2848
39.4	51,11	4,3559	,99	,7119	-,3623	2,1223
39.6	51,04	4,4588	,93	,7226	-,3394	5,8816

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 52

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
26.0	476.40	.1155	1874.04	.4806	-900.2390	.0204
26.2	347.83	.1617	955.33	.4808	-458.7869	.0209
26.4	276.48	.2079	577.40	.4810	-277.2057	.0214
26.6	231.14	.2542	386.15	.4812	-185.3289	.0220
26.8	199.81	.3006	276.17	.4816	-132.4968	.0225
27.0	176.86	.3470	207.08	.4619	-99.2951	.0231
27.2	159.36	.3934	160.99	.4824	-77.1579	.0237
27.4	145.58	.4400	128.64	.4828	-61.6129	.0243
27.6	134.43	.4868	105.03	.4833	-50.2632	.0249
27.8	125.27	.5335	87.37	.4839	-41.7821	.0256
28.0	117.59	.5804	73.75	.4845	-35.2335	.0263
28.2	111.08	.6274	63.05	.4852	-30.0960	.0270
28.4	105.49	.6745	54.51	.4860	-25.9935	.0277
28.6	100.63	.7219	47.52	.4867	-22.6353	.0285
28.8	96.39	.7694	41.80	.4876	-19.8865	.0293
29.0	92.64	.8170	37.01	.4885	-17.5863	.0301
29.2	89.33	.8648	33.00	.4895	-15.6600	.0310
29.4	86.37	.9127	29.58	.4906	-14.0216	.0319
29.6	83.71	.9611	26.64	.4916	-12.6048	.0328
29.8	81.31	1.0096	24.10	.4927	-11.3852	.0338
30.0	79.14	1.0583	21.90	.4939	-10.3259	.0348
30.2	77.17	1.1071	19.97	.4952	-9.4037	.0359
30.4	75.38	1.1561	18.29	.4966	-8.5958	.0370
30.6	73.73	1.2057	16.78	.4980	-7.8700	.0382
30.8	72.22	1.2555	15.44	.4994	-7.2278	.0394
31.0	70.83	1.3054	14.25	.5010	-6.6590	.0407
31.2	69.55	1.3559	13.18	.5026	-6.1441	.0420
31.4	68.37	1.4065	12.22	.5043	-5.6840	.0434
31.6	67.28	1.4574	11.36	.5061	-5.2693	.0449
31.8	66.26	1.5089	10.57	.5079	-4.8915	.0465
32.0	65.32	1.5604	9.86	.5098	-4.5511	.0481
32.2	64.44	1.6132	9.19	.5116	-4.2310	.0499
32.4	63.62	1.6655	8.60	.5138	-3.9490	.0517
32.6	62.86	1.7186	8.06	.5159	-3.6861	.0536
32.8	62.15	1.7719	7.56	.5182	-3.4471	.0556
33.0	61.48	1.8258	7.09	.5205	-3.2261	.0578
33.2	60.85	1.6805	6.66	.5228	-3.0194	.0601
33.4	60.27	1.9354	6.27	.5254	-2.8315	.0625
33.6	59.72	1.9907	5.91	.5280	-2.6585	.0651
33.8	59.21	2.0466	5.57	.5307	-2.4970	.0678
34.0	58.72	2.1036	5.25	.5334	-2.3435	.0707
34.2	58.27	2.1605	4.96	.5364	-2.2056	.0738
34.4	57.84	2.2167	4.68	.5393	-2.0727	.0772
34.6	57.44	2.2770	4.42	.5425	-1.9521	.0808
34.8	57.06	2.3365	4.18	.5457	-1.8369	.0847
35.0	56.70	2.3963	3.96	.5491	-1.7308	.0888
35.2	56.36	2.4570	3.75	.5526	-1.6305	.0933
35.4	56.05	2.5166	3.55	.5562	-1.5363	.0981
35.6	55.75	2.5809	3.36	.5600	-1.4482	.1034
35.8	55.47	2.6438	3.19	.5640	-1.3664	.1090
36.0	55.20	2.7080	3.02	.5680	-1.2877	.1153

N = 52

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
36.2	54.95	2.7726	2,87	,5723	-1,2156	,1221
36.4	54.72	2.8387	2,72	,5767	-1,1460	,1296
36.6	54.49	2,9062	2,58	,5811	-1,0791	,1380
36.8	54.28	2.9742	2,45	,5858	-1,0176	,1472
37.0	54.09	3.0433	2,32	,5908	-,9598	,1575
37.2	53.90	3.1136	2,20	,5960	-,9051	,1689
37.4	53.72	3.1853	2,09	,6013	-,8528	,1820
37.6	53,56	3,2585	1,98	,6068	-,8031	,1969
37,8	53.40	3,3326	1,88	,6126	-,7571	,2138
38.0	53.26	3,4087	1,79	,6185	-,7123	,2337
38,2	53,12	3,4855	1,70	,6249	-,6715	,2564
38,4	52,99	3.5648	1,61	,6314	-,6313	,2842
38.6	52,86	3,6453	1,52	,6382	-,5941	,3172
38,8	52,75	3,7274	1,45	,6454	-,5592	,3574
39,0	52,64	3,8118	1,37	,6529	-,5256	,4087
39.2	52,54	3,8981	1,30	,6607	-,4938	,4751
39,4	52,44	3,9866	1,23	,6688	-,4635	,5655
39,6	52,35	4,0772	1,17	,6774	-,4353	,6931
39,8	52,27	4,1701	1,11	,6864	-,4087	,8869
40.0	52,19	4,2655	1,05	,6958	-,3836	1,2285
40.2	52,12	4,3639	,99	,7056	-,3594	1,9781
40.4	52.05	4,4647	,94	,7160	-,3372	4,7844

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 53

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
26.2	1176.12	.0453	12289,67	,4759	-5848,4652	,0193
26.4	611.31	.0906	3071,63	,4760	-1461,6196	,0198
26.6	416.47	.1359	1364,42	,4761	-649,1327	,0202
26.8	319.16	.1813	767,19	,4763	-364,9527	,0207
27.0	260.78	.2267	490,31	,4765	-233,1318	,0212
27.2	221,95	,2721	340,14	,4768	-161,6733	,0217
27.4	194,26	.3176	249,59	,4771	-118,5853	,0223
27.6	173,53	.3632	190,81	,4775	-90,6149	,0228
27,8	157,43	,4088	150,48	,4779	-71,4161	,0234
28.0	144,60	,4546	121,66	,4784	-57,7039	,0240
28.2	134,12	,5004	100,32	,4789	-47,5435	,0246
28,4	125.42	,5463	84,10	,4795	-39,8262	,0253
28,6	118.08	,5923	71,48	,4801	-33,8230	,0259
28,8	111,60	,6365	61,45	,4808	-29,0452	,0266
29.0	106,39	,6948	53,37	,4815	-25,2037	,0273
29.2	101,68	,7312	46,77	,4823	-22,0631	,0281
29.4	97,53	,7779	41,26	,4831	-19,4401	,0289
29,6	93,87	,8247	36,67	,4840	-17,2572	,0297
29,8	90,61	,8716	32,79	,4850	-15,4097	,0305
30.0	87,68	,9188	29,46	,4859	-13,8249	,0314
30.2	85,06	,9661	26,61	,4870	-12,4663	,0323
30.4	82,69	1.0136	24,14	,4882	-11,2941	,0332
30.6	80,54	1,0612	21,99	,4894	-10,2718	,0342
30.8	78,57	1,1094	20,08	,4906	-9,3649	,0352
31.0	76.78	1.1578	18,40	,4918	-8,5634	,0363
31,2	75,15	1,2060	16,93	,4933	-7,8676	,0374
31.4	73,63	1,2550	15,60	,4947	-7,2348	,0386
31.6	72,25	1,3039	14,43	,4962	-6,6766	,0398
31,8	70,95	1,3536	13,35	,4977	-6,1635	,0411
32.0	69,78	1,4030	12,41	,4994	-5,7156	,0425
32,2	68,68	1.4530	11,54	,5011	-5,3043	,0439
32.4	67,65	1.5034	10,75	,5029	-4,9306	,0454
32.6	66,70	1,5544	10,03	,5046	-4,5860	,0470
32.8	65,82	1.6053	9,38	,5067	-4,2798	,0486
33.0	64,99	1,6569	8,78	,5086	-3,9945	,0503
33.2	64,22	1,7088	8,23	,5107	-3,7332	,0522
33.4	63,49	1,7613	7,72	,5128	-3,4921	,0541
33.6	62,82	1,8140	7,26	,5151	-3,2733	,0562
33.8	62,18	1,8673	6,83	,5174	-3,0691	,0583
34.0	61,59	1,9211	6,43	,5198	-2,8799	,0606
34.2	61,03	1,9752	6,06	,5223	-2,7060	,0630
34.4	60,50	2,0305	5,71	,5248	-2,5400	,0657
34.6	60.01	2,0858	5,40	,5275	-2,3897	,0684
34.8	59,55	2,1414	5,10	,5304	-2,2515	,0713
35.0	59,11	2,1982	4,82	,5332	-2,1184	,0744
35.2	58,70	2,2556	4,56	,5361	-1,9941	,0778
35.4	58,31	2,3129	4,32	,5394	-1,8825	,0813
35.6	57,95	2,3718	4,09	,5425	-1,7731	,0852
35.8	57,60	2,4308	3,87	,5459	-1,6731	,0894
36.0	57,28	2,4906	3,67	,5494	-1,5792	,0938
36.2	56,97	2,5519	3,46	,5529	-1,4881	,0987

N = 53

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
36.4	56,68	2,6130	3,30	,5567	-1,4059	,1039
36.6	56,41	2,6759	3,13	,5605	-1,3252	,1097
36.8	56,15	2,7391	2,97	,5645	-1,2511	,1159
37.0	55,91	2,8030	2,82	,5688	-1,1622	,1227
37.2	55,68	2,8685	2,68	,5730	-1,1151	,1302
37.4	55,46	2,9344	2,55	,5776	-1,0535	,1384
37.6	55,25	3,0020	2,42	,5822	-,9931	,1477
37.8	55,06	3,0703	2,30	,5870	-,9375	,1579
38.0	54,88	3,1399	2,18	,5921	-,8844	,1695
38.2	54,71	3,2102	2,07	,5974	-,8354	,1822
38.4	54,55	3,2822	1,97	,6029	-,7882	,1968
38.6	54,39	3,3557	1,87	,6085	-,7429	,2137
38.8	54,25	3,4304	1,78	,6145	-,7005	,2332
39.0	54,11	3,5066	1,69	,6206	-,6602	,2559
39.2	53,99	3,5847	1,60	,6270	-,6213	,2832
39.4	53,87	3,6639	1,52	,6337	-,5856	,3154
39.6	53,75	3,7452	1,44	,6407	-,5510	,3554
39.8	53,64	3,8284	1,37	,6480	-,5182	,4058
40.0	53,54	3,9131	1,30	,6557	-,4880	,4695
40.2	53,45	4,0001	1,23	,6636	-,4588	,5560
40.4	53,36	4,0893	1,17	,6720	-,4313	,6780
40.6	53,28	4,1809	1,11	,6807	-,4049	,8648
40.8	53,20	4,2748	1,05	,6899	-,3803	1,1806
41.0	53,12	4,3713	1,00	,6995	-,3570	1,8368
41.2	53,05	4,4707	,94	,7095	-,3349	4,0674

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 54

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
27.0	512.71	.1112	2059.30	.4716	-970.6839	.0196
27.2	374.07	.1557	1050.08	.4718	-494.8811	.0201
27.4	297.13	.2002	634.83	.4720	-299.1215	.0205
27.6	248.21	.2448	424.50	.4722	-199.9413	.0210
27.8	214.40	.2894	303.62	.4725	-142.9609	.0215
28.0	189.65	.3341	227.72	.4728	-107.1728	.0221
28.2	170.74	.3788	176.97	.4732	-83.2333	.0226
28.4	155.87	.4236	141.45	.4736	-66.4939	.0232
28.6	143.85	.4686	115.54	.4741	-54.2742	.0238
28.8	133.96	.5135	96.14	.4746	-45.1323	.0244
29.0	125.66	.5587	81.15	.4752	-38.0634	.0250
29.2	118.62	.6039	69.39	.4758	-32.5165	.0256
29.4	112.58	.6492	59.99	.4765	-28.0900	.0263
29.6	107.33	.6947	52.34	.4772	-24.4806	.0270
29.8	102.75	.7403	46.04	.4780	-21.5124	.0277
30.0	98.70	.7860	40.79	.4788	-19.0360	.0285
30.2	95.10	.8321	36.35	.4796	-16.9432	.0292
30.4	91.90	.8781	32.60	.4806	-15.1792	.0301
30.6	89.02	.9243	29.39	.4816	-13.6612	.0309
30.8	86.43	.9708	26.60	.4827	-12.3484	.0318
31.0	84.07	1.0176	24.17	.4837	-11.1996	.0327
31.2	81.93	1.0645	22.05	.4849	-10.2037	.0336
31.4	79.98	1.1116	20.19	.4861	-9.3249	.0346
31.6	78.20	1.1588	18.54	.4874	-8.5527	.0357
31.8	76.55	1.2067	17.07	.4886	-7.8545	.0367
32.0	75.05	1.2544	15.76	.4901	-7.2423	.0379
32.2	73.65	1.3028	14.58	.4915	-6.6850	.0391
32.4	72.37	1.3510	13.54	.4931	-6.1936	.0403
32.6	71.17	1.4000	12.57	.4946	-5.7389	.0416
32.8	70.06	1.4491	11.71	.4962	-5.3314	.0430
33.0	69.04	1.4985	10.92	.4979	-4.9620	.0444
33.2	68.07	1.5465	10.20	.4996	-4.6222	.0459
33.4	67.19	1.5961	9.56	.5017	-4.3211	.0474
33.6	66.35	1.6490	8.95	.5034	-4.0330	.0491
33.8	65.57	1.6996	8.40	.5055	-3.7777	.0509
34.0	64.84	1.7510	7.89	.5076	-3.5384	.0527
34.2	64.15	1.8027	7.43	.5098	-3.3179	.0546
34.4	63.51	1.8551	6.99	.5119	-3.1123	.0567
34.6	62.91	1.9075	6.59	.5143	-2.9258	.0588
34.8	62.34	1.9606	6.22	.5167	-2.7510	.0611
35.0	61.81	2.0143	5.87	.5192	-2.5880	.0636
35.2	61.31	2.0684	5.55	.5218	-2.4371	.0662
35.4	60.83	2.1233	5.24	.5245	-2.2949	.0689
35.6	60.38	2.1788	4.96	.5272	-2.1615	.0719
35.8	59.97	2.2343	4.70	.5302	-2.0407	.0750
36.0	59.57	2.2914	4.45	.5330	-1.9220	.0784
36.2	59.20	2.3481	4.22	.5362	-1.8163	.0819
36.4	58.84	2.4062	4.00	.5393	-1.7130	.0858
36.6	58.51	2.4644	3.79	.5428	-1.6193	.0899
36.8	58.20	2.5239	3.60	.5461	-1.5286	.0944
37.0	57.90	2.5838	3.42	.5497	-1.4443	.0993

N = 54

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
37.2	57.62	2.6450	3,24	,5533	-1,3632	,1046
37.4	57,35	2,7065	3,08	,5572	-1,2889	,1102
37.6	57.10	2,7690	2,93	,5612	-1,2180	,1164
37.8	56,86	2.8325	2,78	,5653	-1,1507	,1233
38.0	56,64	2,8970	2,64	,5696	-1,0872	,1306
38.2	56,43	2,9623	2,51	,5740	-1,0276	,1390
38,4	56.23	3.0784	2,39	,5787	-,9719	,1480
38.6	56,04	3.0963	2,27	,5834	-,9170	,1583
38.8	55,86	3,1649	2,16	,5884	-,8662	,1697
39.0	55,69	3.2345	2,06	,5936	-,8186	,1824
39.2	55.53	3,3059	1,95	,5989	-,7720	,1972
39,4	55,39	3,3760	1,86	,6045	-,7296	,2136
39.6	55,24	3,4521	1.77	,6103	-,6876	,2331
39,8	55.11	3.5273	1,68	,6163	-,6486	,2556
40.0	54,98	3,6039	1,60	,6227	-,6120	,2820
40,2	54,87	3,6822	1,52	,6293	-,5770	,3139
40.4	54,75	3,7623	1,44	,6361	-,5437	,3529
40.6	54,65	3,8441	1,37	,6433	-,5121	,4017
40.8	54,55	3,9279	1,30	,6507	-,4820	,4646
41.0	54,45	4.0137	1,23	,6585	-,4534	,5488
41.2	54,37	4.1015	1,17	,6666	-,4265	,6664
41.4	54,28	4.1913	1,11	,6752	-,4014	,8405
41.6	54,21	4,2837	1,05	,6841	-,3774	1,1333
41.8	54,13	4,3786	1,00	,6935	-,3545	1,7216
42.0	54,06	4,4760	,95	,7033	-,3332	3,4762

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 55

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
27.2	1287.24	.0436	13486.44	.4672	-6300.7309	.0186
27.4	657.37	.0873	3370.81	.4673	-1574.6860	.0190
27.6	447.49	.1310	1497.22	.4674	-699.2888	.0195
27.8	342.65	.1747	841.62	.4675	-392.9991	.0199
28.0	279.81	.2184	538.18	.4678	-251.2332	.0204
28.2	237.95	.2622	373.27	.4680	-174.1811	.0209
28.4	208.12	.3060	273.91	.4683	-127.7665	.0213
28.6	185.77	.3499	209.40	.4686	-97.6248	.0219
28.8	168.45	.3938	165.25	.4690	-77.0084	.0224
29.0	154.59	.4379	133.55	.4694	-62.1936	.0229
29.2	143.31	.4820	110.20	.4699	-51.2895	.0235
29.4	133.92	.5262	92.38	.4704	-42.9604	.0241
29.6	126.00	.5705	78.51	.4710	-36.4805	.0247
29.8	119.23	.6150	67.51	.4716	-31.3399	.0253
30.0	113.39	.6595	58.64	.4722	-27.1972	.0260
30.2	108.30	.7042	51.39	.4730	-23.8094	.0266
30.4	103.83	.7490	45.38	.4737	-21.0019	.0273
30.6	99.87	.7940	40.33	.4745	-18.6445	.0281
30.8	96.35	.8390	36.08	.4754	-16.6600	.0288
31.0	93.20	.8843	32.44	.4764	-14.9601	.0295
31.2	90.35	.9299	29.29	.4773	-13.4906	.0304
31.4	87.78	.9755	26.58	.4783	-12.2256	.0313
31.6	85.45	1.0214	24.21	.4794	-11.1143	.0322
31.8	83.33	1.0674	22.13	.4805	-10.1453	.0331
32.0	81.38	1.1138	20.29	.4816	-9.2838	.0340
32.2	79.61	1.1602	18.67	.4829	-8.5304	.0350
32.4	77.97	1.2070	17.22	.4842	-7.8523	.0361
32.6	76.46	1.2539	15.93	.4856	-7.2503	.0372
32.8	75.07	1.3012	14.76	.4870	-6.7043	.0383
33.0	73.78	1.3487	13.71	.4885	-6.2149	.0395
33.2	72.57	1.3970	12.74	.4899	-5.7633	.0407
33.4	71.47	1.4447	11.89	.4916	-5.3690	.0420
33.6	70.42	1.4937	11.10	.4931	-4.9948	.0434
33.8	69.47	1.5420	10.39	.4950	-4.6687	.0448
34.0	68.56	1.5917	9.72	.4967	-4.3554	.0463
34.2	67.72	1.6407	9.13	.4987	-4.0814	.0479
34.4	66.93	1.6909	8.57	.5005	-3.8193	.0496
34.6	66.19	1.7411	8.06	.5026	-3.5821	.0513
34.8	65.50	1.7917	7.59	.5047	-3.3638	.0532
35.0	64.84	1.8432	7.15	.5067	-3.1567	.0551
35.2	64.23	1.8947	6.74	.5090	-2.9691	.0572
35.4	63.65	1.9468	6.37	.5113	-2.7933	.0594
35.6	63.11	1.9990	6.02	.5138	-2.6334	.0617
35.8	62.60	2.0525	5.69	.5162	-2.4782	.0641
36.0	62.12	2.1057	5.39	.5188	-2.3393	.0667
36.2	61.66	2.1600	5.10	.5215	-2.2056	.0695
36.4	61.23	2.2147	4.83	.5242	-2.0811	.0724
36.6	60.83	2.2698	4.58	.5271	-1.9659	.0756
36.8	60.45	2.3257	4.35	.5300	-1.8567	.0789
37.0	60.09	2.3824	4.12	.5331	-1.7536	.0825
37.2	59.74	2.4397	3.91	.5362	-1.6568	.0864

N = 55

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
37.4	59.42	2.4976	3,72	,5395	-1,5663	,0906
37.6	59,12	2.5559	3,53	,5430	-1,4824	,0950
37.8	58,83	2,6153	3,36	,5465	-1,4016	,0998
38.0	58,56	2,6755	3,19	,5501	-1,3254	,1051
38.2	58,30	2.7363	3,04	,5540	-1,2543	,1107
38.4	58,06	2.7978	2,89	,5580	-1,1875	,1169
38.6	57,82	2.8611	2,74	,5619	-1,1214	,1238
38.8	57.61	2.9242	2,61	,5663	-1,0624	,1311
39.0	57,40	2,9891	2,49	,5706	-1,0040	,1394
39.2	57.21	3.0546	2,37	,5751	-,9498	,1484
39.4	57,02	3.1216	2,25	,5798	-,8973	,1587
39.6	56,84	3,1894	2,14	,5847	-,8483	,1700
39.8	56,68	3.2579	2,04	,5899	-,8029	,1825
40.0	56,53	3,3281	1,94	,5951	-,7587	,1969
40.2	56,38	3,3997	1,85	,6006	-,7167	,2135
40.4	56,24	3,4725	1,76	,6063	-,6769	,2325
40.6	56.11	3.5470	1,67	,6122	-,6385	,2549
40.8	55,98	3,6227	1,59	,6184	-,6026	,2811
41.0	55,86	3,7001	1,51	,6248	-,5683	,3127
41.2	55,76	3,7787	1,44	,6316	-,5367	,3504
41.4	55.65	3,8595	1,37	,6386	-,5056	,3984
41.6	55,55	3,9420	1,30	,6459	-,4765	,4594
41.8	55,46	4,0264	1,23	,6535	-,4489	,5400
42.0	55,37	4,1129	1,17	,6615	-,4227	,6525
42.2	55,29	4,2016	1,11	,6698	-,3977	,8157
42.4	55,21	4,2925	1,06	,6785	-,3742	1,0929
42.6	55,14	4,3858	1,00	,6876	-,3520	1,6220
42.8	55,07	4,4817	,95	,6972	-,3309	3,0916
43.0	55,01	4,5801	,90	,7072	-,3112	26,0159

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 56

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
28.0	550.43	,1072	2256,19	,4631	-1044,4417	,0189
28.2	401.27	,1501	1150,25	,4632	-532,3402	,0193
28.4	318,47	,1931	695,16	,4634	-321,6234	,0197
28.6	265,88	,2360	465,03	,4636	-215,1026	,0202
28.8	229,51	,2790	332,64	,4639	-153,8161	,0207
29.0	202,88	,3221	249,54	,4642	-115,3440	,0211
29.2	182,53	,3652	193,95	,4645	-89,5959	,0216
29.4	166,51	,4084	154,98	,4649	-71,5529	,0222
29,6	153,58	,4517	126,64	,4653	-58,4305	,0227
29,8	142,91	,4951	105,34	,4658	-48,5668	,0232
30.0	133,99	,5385	88,97	,4663	-40,9937	,0238
30.2	126,42	,5820	76,11	,4669	-35,0420	,0244
30.4	119,90	,6257	65,79	,4675	-30,2585	,0250
30.6	114,25	,6695	57,42	,4682	-26,3886	,0256
30,8	109,30	,7134	50,52	,4689	-23,1924	,0263
31.0	104,94	,7574	44,77	,4697	-20,5333	,0270
31,2	101,06	,8017	39,91	,4704	-18,2823	,0277
31,4	97,60	,8460	35,80	,4713	-16,3791	,0284
31,6	94,49	,8904	32,28	,4722	-14,7500	,0292
31,8	91,69	,9350	29,23	,4732	-13,3403	,0300
32,0	89,15	,9799	26,58	,4742	-12,1112	,0308
32,2	86,84	1,0249	24,26	,4752	-11,0382	,0316
32,4	84,72	1,0704	22,20	,4762	-10,0851	,0325
32,6	82,79	1,1158	20,40	,4774	-9,2531	,0335
32,8	81,01	1,1615	18,79	,4786	-8,5078	,0344
33.0	79,39	1,2073	17,37	,4799	-7,8501	,0354
33,2	77,87	1,2537	16,07	,4812	-7,2484	,0365
33,4	76,47	1,3001	14,91	,4825	-6,7142	,0376
33,6	75,19	1,3465	13,88	,4841	-6,2373	,0387
33,8	73,99	1,3935	12,93	,4855	-5,7984	,0399
34.0	72,86	1,4409	12,07	,4871	-5,3985	,0412
34,2	71,83	1,4884	11,29	,4887	-5,0379	,0425
34,4	70,84	1,5367	10,56	,4902	-4,6988	,0438
34,6	69,94	1,5047	9,91	,4921	-4,3996	,0453
34,8	69,09	1,6331	9,30	,4939	-4,1227	,0468
35.0	68,29	1,6823	8,74	,4957	-3,8623	,0484
35,2	67,54	1,7318	8,22	,4976	-3,6230	,0501
35,4	66,84	1,7813	7,75	,4997	-3,4070	,0518
35,6	66,18	1,8315	7,31	,5017	-3,2024	,0537
35,8	65,55	1,8821	6,90	,5039	-3,0136	,0556
36.0	64,97	1,9329	6,53	,5062	-2,8406	,0577
36,2	64,42	1,9845	6,17	,5085	-2,6761	,0599
36,4	63,90	2,0364	5,84	,5109	-2,5241	,0622
36,6	63,41	2,0890	5,53	,5133	-2,3810	,0646
36,8	62,94	2,1422	5,24	,5158	-2,2470	,0672
37.0	62,50	2,1957	4,97	,5185	-2,1223	,0700
37,2	62,09	2,2495	4,71	,5213	-2,0071	,0729
37,4	61,70	2,3041	4,47	,5241	-1,8980	,0761
37,6	61,34	2,3593	4,25	,5271	-1,7950	,0794
37,8	60,99	2,4152	4,04	,5302	-1,6983	,0830
38,0	60,66	2,4715	3,84	,5334	-1,6082	,0869

N = 56

RATIC	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
38.2	60.34	2.5289	3,65	,5366	-1,5212	,0910
38.4	60.05	2.5868	3,47	,5400	-1,4401	,0955
38.6	59,77	2,6455	3,30	,5435	-1,3633	,1003
38.8	59,50	2,7049	3,14	,5471	-1,2910	,1055
39.0	59,25	2.7654	2,99	,5508	-1,2215	,1112
39.2	59,01	2,8267	2,85	,5546	-1,1558	,1175
39.4	58,79	2,8887	2,71	,5586	-1,0941	,1243
39.6	58,57	2,9514	2,58	,5628	-1,0364	,1316
39.8	58,37	3,0150	2,46	,5672	-,9821	,1397
40.0	58,18	3.0799	2,34	,5717	-,9295	,1488
40.2	58.00	3.1462	2,23	,5763	-,8767	,1590
40.4	57,83	3.2131	2,12	,5811	-,8315	,1702
40.6	57,67	3,2812	2,02	,5861	-,7868	,1828
40.8	57,51	3,3505	1,93	,5913	-,7442	,1971
41,0	57,37	3,4208	1,84	,5968	-,7043	,2133
41.2	57,23	3,4926	1,75	,6024	-,6659	,2320
41.4	57,10	3.5660	1,66	,6082	-,6290	,2540
41.6	56,98	3.6409	1,58	,6142	-,5938	,2800
41.8	56,87	3.7169	1,51	,6206	-,5612	,3105
42.0	56,76	3,7948	1,44	,6272	-,5296	,3481
42,2	56,65	3,8742	1,37	,6341	-,4999	,3943
42.4	56,56	3,9556	1,30	,6412	-,4714	,4538
42,6	56.47	4,0387	1,24	,6487	-,4445	,5317
42.8	56,38	4,1239	1,17	,6564	-,4190	,6391
43.0	56,30	4,2114	1,12	,6645	-,3944	,7985
43,2	56,22	4,3006	1,06	,6731	-,3716	1,0507
43,4	56,15	4,3926	1,01	,6819	-,3496	1,5312
43,6	56,08	4,4868	,96	,6912	-,3292	2,7456
43,8	56,02	4,5837	,91	,7010	-,3096	12,7656

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 57

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
28.2	1381.47	.0421	14743.30	.4589	-6765.4178	.0179
28.4	704.99	.0842	3685.01	.4590	-1690.8573	.0183
28.6	479.57	.1264	1636.88	.4591	-750.9398	.0187
28.8	366.97	.1686	920.22	.4592	-422.0807	.0191
29.0	299.44	.2108	588.37	.4594	-269.7834	.0196
29.2	254.51	.2530	408.26	.4596	-187.1490	.0200
29.4	222.45	.2953	299.60	.4599	-137.2834	.0205
29.6	198.46	.3376	229.13	.4602	-104.9587	.0209
29.8	179.81	.3800	180.73	.4606	-82.7414	.0214
30.0	164.92	.4225	146.10	.4609	-66.8433	.0219
30.2	152.78	.4650	120.54	.4614	-55.1179	.0225
30.4	142.68	.5077	101.05	.4618	-46.1696	.0230
30.6	134.19	.5503	85.96	.4624	-39.2515	.0235
30.8	126.91	.5931	73.93	.4630	-33.7331	.0241
31.0	120.62	.6360	64.23	.4636	-29.2765	.0247
31.2	115.13	.6791	56.28	.4642	-25.6308	.0253
31.4	110.32	.7223	49.71	.4649	-22.6147	.0260
31.6	106.06	.7655	44.21	.4657	-20.0919	.0266
31.8	102.26	.8089	39.54	.4664	-17.9504	.0273
32.0	98.85	.8526	35.55	.4672	-16.1159	.0280
32.2	95.79	.8964	32.12	.4681	-14.5456	.0288
32.4	93.02	.9403	29.16	.4690	-13.1841	.0295
32.6	90.50	.9844	26.56	.4700	-11.9927	.0303
32.8	88.22	1.0285	24.30	.4711	-10.9592	.0311
33.0	86.12	1.0731	22.29	.4721	-10.0349	.0320
33.2	84.20	1.1177	20.51	.4733	-9.2213	.0329
33.4	82.43	1.1625	18.93	.4745	-8.4958	.0338
33.6	80.79	1.2079	17.50	.4756	-7.8373	.0348
33.8	79.29	1.2531	16.23	.4770	-7.2573	.0358
34.0	77.88	1.2990	15.07	.4782	-6.7248	.0369
34.2	76.59	1.3448	14.04	.4796	-6.2507	.0380
34.4	75.39	1.3906	13.10	.4812	-5.8250	.0391
34.6	74.26	1.4371	12.24	.4826	-5.4292	.0403
34.8	73.21	1.4842	11.45	.4841	-5.0641	.0416
35.0	72.23	1.5310	10.74	.4858	-4.7391	.0429
35.2	71.31	1.5784	10.07	.4875	-4.4365	.0443
35.4	70.45	1.6263	9.47	.4891	-4.1567	.0457
35.6	69.64	1.6744	8.91	.4909	-3.9003	.0472
35.8	68.89	1.7227	8.39	.4928	-3.6651	.0488
36.0	68.18	1.7714	7.91	.4948	-3.4474	.0505
36.2	67.51	1.8205	7.47	.4968	-3.2454	.0523
36.4	66.88	1.8698	7.06	.4990	-3.0592	.0541
36.6	66.29	1.9201	6.67	.5011	-2.8814	.0561
36.8	65.73	1.9704	6.32	.5034	-2.7199	.0581
37.0	65.20	2.0213	5.98	.5057	-2.5672	.0603
37.2	64.70	2.0728	5.67	.5080	-2.4237	.0627
37.4	64.23	2.1243	5.39	.5104	-2.2894	.0651
37.6	63.78	2.1772	5.10	.5130	-2.1645	.0677
37.8	63.36	2.2293	4.85	.5157	-2.0492	.0705
38.0	62.97	2.2831	4.61	.5185	-1.9400	.0734
38.2	62.58	2.3376	4.37	.5212	-1.8336	.0766

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UNCLASSIFIED MDC-G6553 AFOSR-TR-77-0046 NL

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N = 57

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
38.4	62.23	2.3920	4.16	.5242	-1.7371	.0799
38.6	61.89	2.4476	3.95	.5271	-1.6437	.0836
38.8	61.57	2.5035	3.76	.5303	-1.5571	.0874
39.0	61.27	2.5596	3.58	.5336	-1.4772	.0915
39.2	60.98	2.6173	3.41	.5369	-1.3983	.0960
39.4	60.71	2.6754	3.25	.5403	-1.3248	.1009
39.6	60.45	2.7341	3.09	.5439	-1.2558	.1061
39.8	60.20	2.7935	2.95	.5477	-1.1907	.1117
40.0	59.97	2.8536	2.81	.5516	-1.1296	.1178
40.2	59.75	2.9153	2.68	.5555	-1.0692	.1246
40.4	59.54	2.9776	2.55	.5595	-1.0129	.1321
40.6	59.35	3.0403	2.43	.5639	-.9610	.1401
40.8	59.16	3.1045	2.32	.5683	-.9100	.1491
41.0	58.98	3.1698	2.21	.5728	-.8615	.1592
41.2	58.81	3.2360	2.11	.5776	-.8158	.1703
41.4	58.66	3.3030	2.01	.5826	-.7732	.1827
41.6	58.51	3.3715	1.92	.5877	-.7319	.1969
41.8	58.36	3.4414	1.83	.5929	-.6921	.2132
42.0	58.23	3.5124	1.74	.5985	-.6547	.2318
42.2	58.10	3.5847	1.66	.6042	-.6194	.2534
42.4	57.98	3.6585	1.58	.6101	-.5854	.2789
42.6	57.87	3.7337	1.50	.6163	-.5532	.3092
42.8	57.76	3.8103	1.43	.6228	-.5230	.3456
43.0	57.65	3.8868	1.36	.6296	-.4939	.3911
43.2	57.56	3.9688	1.30	.6366	-.4664	.4484
43.4	57.47	4.0508	1.24	.6439	-.4402	.5237
43.6	57.39	4.1348	1.18	.6515	-.4149	.6277
43.8	57.31	4.2206	1.12	.6595	-.3915	.7761
44.0	57.23	4.3088	1.06	.6677	-.3688	1.0146
44.2	57.16	4.3990	1.01	.6764	-.3477	1.4440
44.4	57.09	4.4919	.96	.6854	-.3273	2.4851
44.6	57.03	4.5871	.91	.6949	-.3082	8.3549

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between *N* and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 59

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
29.0	589.35	.1035	2462.10	.4550	-1119.7692	.0182
29.2	429.42	.1449	1255.93	.4552	-571.1641	.0186
29.4	340.61	.1864	759.22	.4553	-345.1895	.0190
29.6	284.14	.2279	507.71	.4555	-230.7611	.0194
29.8	245.13	.2694	363.25	.4558	-165.0625	.0198
30.0	216.52	.3110	272.43	.4560	-123.7327	.0203
30.2	194.72	.3525	211.86	.4564	-96.1856	.0207
30.4	177.53	.3942	169.35	.4567	-76.8498	.0212
30.6	163.62	.4360	138.34	.4571	-62.7323	.0217
30.8	152.18	.4778	115.12	.4575	-52.1689	.0222
31.0	142.61	.5197	97.27	.4580	-44.0586	.0227
31.2	134.46	.5617	83.20	.4585	-37.6533	.0233
31.4	127.45	.6039	71.92	.4591	-32.5189	.0238
31.6	121.37	.6461	62.77	.4597	-28.3593	.0244
31.8	116.06	.6883	55.26	.4604	-24.9442	.0250
32.0	111.36	.7308	48.96	.4610	-22.0782	.0256
32.2	107.20	.7733	43.69	.4618	-19.6790	.0263
32.4	103.48	.8160	39.19	.4625	-17.6345	.0269
32.6	100.12	.8590	35.32	.4633	-15.8712	.0276
32.8	97.10	.9020	32.00	.4642	-14.3612	.0284
33.0	94.37	.9451	29.11	.4651	-13.0489	.0291
33.2	91.83	.9884	26.57	.4661	-11.8958	.0299
33.4	89.60	1.0321	24.34	.4670	-10.8777	.0307
33.6	87.52	1.0757	22.37	.4681	-9.9831	.0315
33.8	85.61	1.1196	20.62	.4692	-9.1887	.0324
34.0	83.84	1.1638	19.05	.4703	-8.4724	.0333
34.2	82.21	1.2082	17.64	.4715	-7.8353	.0342
34.4	80.69	1.2529	16.37	.4727	-7.2563	.0352
34.6	79.30	1.2974	15.25	.4741	-6.7462	.0362
34.8	78.00	1.3426	14.21	.4755	-6.2748	.0372
35.0	76.78	1.3881	13.26	.4768	-5.8431	.0384
35.2	75.65	1.4338	12.40	.4782	-5.4516	.0395
35.4	74.60	1.4795	11.63	.4798	-5.1005	.0407
35.6	73.61	1.5259	10.90	.4813	-4.7716	.0420
35.8	72.69	1.5722	10.25	.4830	-4.4745	.0433
36.0	71.83	1.6189	9.64	.4848	-4.2006	.0447
36.2	71.01	1.6664	9.07	.4864	-3.9416	.0461
36.4	70.24	1.7137	8.56	.4883	-3.7085	.0477
36.6	69.52	1.7617	8.07	.4901	-3.4890	.0493
36.8	68.85	1.8097	7.63	.4921	-3.2895	.0509
37.0	68.21	1.8586	7.21	.4941	-3.0982	.0527
37.2	67.60	1.9076	6.83	.4961	-2.9233	.0546
37.4	67.04	1.9570	6.47	.4983	-2.7610	.0565
37.6	66.50	2.0069	6.13	.5005	-2.6077	.0586
37.8	66.00	2.0570	5.82	.5029	-2.4673	.0608
38.0	65.52	2.1079	5.52	.5053	-2.3326	.0631
38.2	65.06	2.1592	5.24	.5078	-2.2075	.0656
38.4	64.63	2.2112	4.98	.5102	-2.0885	.0682
38.6	64.23	2.2632	4.73	.5129	-1.9793	.0710
38.8	63.84	2.3165	4.50	.5156	-1.8729	.0739
39.0	63.48	2.3696	4.28	.5185	-1.7766	.0771

N = 58

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
39.2	63,13	2,4238	4,08	,5213	-1,6834	,0804
39.4	62,80	2,4782	3,88	,5244	-1,5970	,0840
39.6	62,49	2,5336	3,70	,5275	-1,5141	,0879
39.8	62.19	2,5899	3,52	,5306	-1,4347	,0920
40.0	61,92	2,6461	3,36	,5340	-1,3624	,0964
40.2	61,65	2,7040	3,20	,5374	-1,2905	,1013
40.4	61.40	2,7616	3,05	,5410	-1,2259	,1064
40.6	61.16	2,8209	2,91	,5446	-1,1620	,1121
40.8	60,93	2,8806	2,78	,5484	-1,1022	,1183
41.0	60.72	2,9411	2,65	,5524	-1,0457	,1250
41.2	60.52	3.0025	2,53	,5564	-,9919	,1323
41.4	60,32	3,0649	2,41	,5606	-,9408	,1409
41.6	60.14	3.1284	2,30	,5650	-,8916	,1494
41.8	59,96	3,1930	2,19	,5694	-,8446	,1594
42.0	59,80	3,2581	2,09	,5742	-,8013	,1703
42.2	59,65	3,3245	2,00	,5790	-,7595	,1827
42.4	59,50	3,3923	1,90	,5840	-,7192	,1969
42.6	59,36	3,4611	1,82	,5893	-,6813	,2128
42.8	59,23	3,5310	1,73	,5947	-,6454	,2310
43.0	59.10	3,6027	1,65	,6003	-,6104	,2525
43.2	58,98	3,6754	1,57	,6062	-,5777	,2775
43.4	58,87	3,7494	1,50	,6123	-,5469	,3069
43.6	58,76	3,8254	1,43	,6186	-,5167	,3431
43.8	58,66	3,9026	1,36	,6252	-,4867	,3871
44.0	58,57	3,9816	1,30	,6321	-,4616	,4431
44.2	58,48	4,0625	1,24	,6392	-,4359	,5160
44.4	58,39	4,1452	1,18	,6466	-,4114	,6153
44.6	58,31	4,2297	1,12	,6544	-,3884	,7568
44.8	58,24	4,3164	1,07	,6626	-,3666	,9768
45.0	58,17	4,4055	1,02	,6710	-,3454	1,3716
45.2	58,10	4,4967	,97	,6798	-,3256	2,2638
45.4	58,03	4,5904	,92	,6890	-,3068	6,2316

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 59

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
29.2	1479.26	,0407	16075,64	,4511	-7251,2774	,0173
29.4	754.27	,0814	4016,17	,4511	-1011,1695	,0177
29.6	512,82	,1221	1784,71	,4512	-804,8155	,0180
29.8	392,11	,1628	1003,05	,4513	-452,1975	,0184
30.0	319,80	,2036	641,64	,4515	-289,2209	,0189
30.2	271,62	,2444	445,13	,4517	-200,5770	,0192
30.4	237,25	,2852	326,66	,4520	-147,1360	,0197
30.6	211.53	,3261	249,81	,4523	-112,4789	,0201
30.8	191,56	,3670	197,16	,4526	-88,7375	,0206
31.0	175.60	,4080	159,42	,4530	-71,7114	,0210
31.2	162,58	,4491	131,53	,4534	-59,1294	,0215
31.4	151,75	,4902	110,29	,4538	-49,5472	,0220
31.6	142.61	,5315	93,77	,4542	-42,0932	,0225
31.8	134,80	,5728	80,67	,4547	-36,1851	,0230
32.0	128,06	,6142	70,11	,4553	-31,4238	,0236
32.2	122.18	,6557	61,47	,4559	-27,5291	,0241
32.4	117.01	,6973	54,30	,4566	-24,2958	,0247
32.6	112,43	,7390	48,29	,4572	-21,5845	,0253
32.8	108,34	,7810	43,18	,4579	-19,2801	,0259
33.0	104,70	,8229	38,86	,4587	-17,3356	,0266
33.2	101,40	,8651	35,12	,4595	-15,6455	,0273
33.4	98,42	,9074	31,88	,4603	-14,1834	,0280
33.6	95,71	,9499	29,05	,4612	-12,9085	,0287
33.8	93,25	,9925	26,58	,4622	-11,7951	,0294
34.0	90,98	1,0354	24,39	,4631	-10,8059	,0302
34.2	88,91	1,0784	22,44	,4642	-9,9297	,0310
34,4	87,01	1,1215	20,72	,4653	-9,1551	,0318
34.6	85,25	1,1648	19,18	,4664	-8,4597	,0327
34.8	83,62	1,2084	17,79	,4676	-7,8334	,0336
35.0	82,11	1,2523	16,53	,4688	-7,2660	,0345
35.2	80.71	1,2963	15,40	,4701	-6,7561	,0355
35.4	79,40	1,3408	14,37	,4713	-6,2898	,0366
35,6	78,19	1,3852	13,44	,4727	-5,8717	,0376
35.8	77,06	1,4301	12,58	,4741	-5,4845	,0387
36.0	75,99	1,4754	11,79	,4755	-5,1289	,0399
36.2	75,00	1,5208	11,07	,4770	-4,8054	,0411
36.4	74.07	1,5661	10,42	,4788	-4,5138	,0424
36.6	73.20	1,6122	9,81	,4803	-4,2370	,0437
36.8	72,37	1,6585	9,24	,4820	-3,9841	,0451
37.0	71,60	1,7052	8,72	,4838	-3,7490	,0465
37.2	70,87	1,7522	8,24	,4856	-3,5318	,0481
37.4	70,19	1,7994	7,79	,4875	-3,3308	,0497
37.6	69,54	1,8472	7,37	,4894	-3,1422	,0514
37.8	68,93	1,8954	6,98	,4914	-2,9662	,0531
38.0	68,35	1,9439	6,62	,4935	-2,8030	,0550
38.2	67,80	1,9929	6,27	,4956	-2,6491	,0570
38.4	67,29	2,0420	5,96	,4979	-2,5082	,0590
38.6	66.80	2.0919	5,66	,5002	-2,3731	,0613
38.8	66,34	2.1421	5,38	,5026	-2,2478	,0636
39.0	65.90	2.1930	5,11	,5050	-2,1265	,0661
39.2	65.49	2,2440	4,86	,5076	-2,0193	,0686

N = 59

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
39.4	65.10	2.2960	4,63	,5102	-1,9129	,0714
39.6	64,72	2,3485	4,40	,5128	-1,8132	,0744
39.8	64,37	2,4014	4,20	,5156	-1,7202	,0776
40.0	64,04	2,4545	4,00	,5186	-1,6341	,0809
40.2	63,71	2,5092	3,81	,5214	-1,5460	,0845
40.4	63,42	2,5632	3,64	,5247	-1,4724	,0883
40.6	63.13	2,6187	3,47	,5278	-1,3971	,0925
40.8	62,86	2,6749	3,31	,5311	-1,3257	,0969
41.0	62,60	2,7318	3,15	,5344	-1,2562	,1018
41.2	62,35	2,7893	3,01	,5380	-1,1948	,1069
41.4	62,12	2,8472	2,87	,5417	-1,1355	,1125
41.6	61,90	2,9066	2,74	,5454	-1,0771	,1187
41.8	61,69	2,9663	2,62	,5493	-1,0230	,1253
42.0	61,49	3,0275	2,50	,5532	-,9700	,1327
42.2	61,30	3,0888	2,39	,5574	-,9215	,1407
42.4	61,12	3,1515	2,28	,5617	-,8743	,1496
42.6	60,95	3,2153	2,18	,5661	-,8292	,1595
42.8	60,79	3,2796	2,08	,5708	-,7872	,1704
43.0	60,64	3,3453	1,98	,5756	-,7468	,1826
43.2	60,49	3,4124	1,89	,5805	-,7074	,1967
43.4	60,35	3,4804	1,81	,5856	-,6706	,2125
43.6	60,22	3,5494	1,73	,5910	-,6359	,2304
43.8	60,10	3,6200	1,65	,5965	-,6022	,2515
44.0	59,98	3,6918	1,57	,6023	-,5703	,2760
44.2	59,87	3,7652	1,50	,6082	-,5396	,3054
44.4	59,77	3,8398	1,43	,6145	-,5109	,3403
44.6	59,67	3,9161	1,36	,6210	-,4835	,3833
44.8	59,57	3,9941	1,30	,6276	-,4569	,4380
45.0	59,48	4,0737	1,24	,6346	-,4320	,5082
45.2	59,40	4,1553	1,18	,6419	-,4081	,6034
45.4	59,32	4,2386	1,13	,6495	-,3855	,7382
45.6	59,24	4,3241	1,07	,6574	-,3638	,9462
45.8	59,17	4,4116	1,02	,6657	-,3434	1,3046
46.0	59,11	4,5014	,97	,6743	-,3240	2,0737
46.2	59,04	4,5935	,92	,6833	-,3056	4,9350

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

N = 60

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
30.0	629.71	.1000	2680,47	,4474	-1198,6596	,0175
30.2	458.53	.1401	1367,24	,4475	-611,3529	,0179
30.4	363.49	,1801	826,68	,4477	-369,5852	,0183
30.6	303.02	,2203	552,74	,4478	-247,0182	,0187
30.8	261.25	,2604	395,48	,4481	-176,6999	,0191
31.0	230.64	,3005	296,66	,4483	-132,4880	,0195
31.2	207.29	,3407	230,72	,4486	-103,0024	,0199
31.4	188.86	,3810	184,38	,4489	-82,2666	,0204
31.6	174.01	,4213	150,75	,4493	-67,2359	,0208
31.8	161.73	,4618	125,41	,4497	-55,8923	,0213
32.0	151.45	,5023	105,91	,4501	-47,1669	,0218
32.2	142.74	,5428	90,65	,4506	-40,3505	,0223
32.4	135.24	,5834	78,40	,4511	-34,8731	,0228
32.6	128.72	,6242	68,43	,4516	-30,4121	,0233
32.8	123.02	,6650	60,25	,4523	-26,7514	,0239
33.0	117.99	,7060	53,40	,4529	-23,6879	,0244
33.2	113.52	,7470	47,65	,4536	-21,1184	,0250
33.4	109.52	,7882	42,75	,4543	-18,9277	,0256
33.6	105.92	,8297	38,54	,4549	-17,0396	,0262
33.8	102.66	,8711	34,92	,4558	-15,4250	,0269
34.0	99.73	,9128	31,77	,4566	-14,0126	,0276
34.2	97.07	,9544	29,03	,4575	-12,7896	,0283
34.4	94.62	,9963	26,60	,4584	-11,7037	,0290
34.6	92.38	1,0384	24,45	,4594	-10,7439	,0297
34.8	90.31	1,0808	22,53	,4604	-9,8865	,0305
35.0	88.41	1,1234	20,82	,4614	-9,1207	,0313
35.2	86.65	1,1661	19,29	,4624	-8,4357	,0322
35.4	85.03	1,2090	17,92	,4636	-7,8208	,0330
35.6	83.52	1,2521	16,67	,4647	-7,2657	,0340
35.8	82.13	1,2952	15,56	,4661	-6,7706	,0349
36.0	80.81	1,3390	14,53	,4673	-6,3057	,0359
36.2	79.59	1,3827	13,60	,4686	-5,8916	,0369
36.4	78.45	1,4269	12,74	,4700	-5,5089	,0380
36.6	77.38	1,4713	11,96	,4714	-5,1584	,0391
36.8	76.38	1,5158	11,24	,4729	-4,8402	,0403
37.0	75.45	1,5606	10,58	,4744	-4,5455	,0415
37.2	74.57	1,6057	9,97	,4761	-4,2746	,0428
37.4	73.73	1,6514	9,40	,4776	-4,0194	,0441
37.6	72.96	1,6968	8,89	,4795	-3,7906	,0455
37.8	72.22	1,7431	8,40	,4812	-3,5716	,0469
38.0	71.52	1,7897	7,94	,4830	-3,3692	,0485
38.2	70.87	1,8364	7,53	,4849	-3,1833	,0501
38.4	70.25	1,8838	7,13	,4868	-3,0063	,0518
38.6	69.67	1,9310	6,77	,4889	-2,8461	,0535
38.8	69.11	1,9792	6,42	,4909	-2,6914	,0554
39.0	68.59	2,0279	6,10	,4930	-2,5463	,0574
39.2	68.10	2,0763	5,80	,4953	-2,4145	,0595
39.4	67.63	2,1255	5,51	,4976	-2,2888	,0617
39.6	67.18	2,1754	5,24	,5000	-2,1694	,0640
39.8	66.76	2,2258	4,99	,5024	-2,0565	,0665
40.0	66.35	2,2767	4,75	,5048	-1,9501	,0691

N = 60

RATIO	TOTAL ERROR	STEP SIZE	VAR ERROR	VAR STEP	COVAR	MTTF
40.2	65.97	2.3280	4.53	.5074	-1.8505	.0719
40.4	65.61	2.3796	4.32	.5102	-1.7577	.0749
40.6	65.27	2.4321	4.11	.5129	-1.6683	.0780
40.8	64.95	2.4847	3.93	.5158	-1.5859	.0813
41.0	64.64	2.5381	3.75	.5188	-1.5072	.0849
41.2	64.35	2.5921	3.58	.5219	-1.4323	.0887
41.4	64.07	2.6470	3.41	.5250	-1.3604	.0929
41.6	63.80	2.7030	3.26	.5282	-1.2908	.0974
41.8	63.55	2.7588	3.11	.5316	-1.2279	.1021
42.0	63.31	2.8160	2.97	.5350	-1.1658	.1074
42.2	63.08	2.8736	2.84	.5386	-1.1079	.1130
42.4	62.87	2.9317	2.71	.5424	-1.0536	.1190
42.6	62.66	2.9909	2.59	.5462	-1.0012	.1257
42.8	62.46	3.0512	2.48	.5502	-.9508	.1330
43.0	62.28	3.1121	2.37	.5543	-.9033	.1410
43.2	62.10	3.1740	2.26	.5585	-.8581	.1498
43.4	61.94	3.2367	2.16	.5630	-.8151	.1595
43.6	61.78	3.3010	2.06	.5674	-.7727	.1706
43.8	61.63	3.3655	1.97	.5722	-.7345	.1825
44.0	61.48	3.4319	1.88	.5770	-.6962	.1965
44.2	61.35	3.4988	1.80	.5821	-.6610	.2119
44.4	61.22	3.5673	1.72	.5874	-.6266	.2299
44.6	61.10	3.6371	1.64	.5928	-.5938	.2507
44.8	60.98	3.7077	1.57	.5985	-.5633	.2745
45.0	60.87	3.7801	1.50	.6043	-.5336	.3032
45.2	60.77	3.8540	1.43	.6104	-.5051	.3378
45.4	60.67	3.9292	1.36	.6168	-.4783	.3798
45.6	60.58	4.0062	1.30	.6233	-.4525	.4328
45.8	60.49	4.0847	1.24	.6302	-.4281	.5008
46.0	60.41	4.1651	1.18	.6373	-.4048	.5920
46.2	60.33	4.2472	1.13	.6447	-.3826	.7204
46.4	60.25	4.3313	1.08	.6525	-.3616	.9137
46.6	60.18	4.4175	1.02	.6605	-.3415	1.2425
46.8	60.12	4.5061	.98	.6689	-.3222	1.9238
47.0	60.05	4.5966	.93	.6777	-.3043	4.0970

Column 1 is the ratio $\Sigma(i-1)X_i/\Sigma X_i$

Column 2 is the estimate for the total error content

Column 3 is the normed estimate for step size: in order to determine the actual estimate for the step size, the entry in this column should be divided by the total observation time T.

Column 4 is the approximate standard deviation of the estimate of the total error content.

Column 5 is the normed standard deviation of the estimate of the step size: in order to obtain the actual standard deviation the entry in this column should be divided by the total time T.

Column 6 is the normed covariance between N and ϕ: in order to obtain the actual estimated covariance the entry should be divided by T.

Column 7 is the normed MTTF and in order to obtain the actual value the entry should be multiplied by T.

Appendix III
FORMULAS FOR THE GEOMETRIC DE-EUTROPHICATION PROCESS

Summarized here are formulas for this process which can be evaluated when the two parameters D and k are found. The parameter can be solved by the equation:

$$\frac{\Sigma i k^{i-1} X_i}{\Sigma k^{i-1} X_i} = \frac{n+1}{2} \qquad \text{III-1}$$

With this value of $k(\hat{k})$, the value of D can be determined through the equation:

$$\hat{D} = \frac{n}{\hat{k}^{i-1} X_i} \qquad \text{III-2}$$

The estimate for the MTTF at the end of test is

$$M_2 = \hat{D}_k^{\hat{}n} \; {}^{-1} \qquad \text{III-3}$$

The estimate of purification percentage is

$$\hat{P}_2 = (1-\hat{k}^n)100 \qquad \text{III-4}$$

The variance and covariances are given by

$$\text{Var } \hat{D} = D^2 \; \frac{2(2n-1)}{n(n+1)} \qquad \text{III-5}$$

$$\text{Var } \hat{k} = k^2 \; \frac{12}{n(n^2-1)} \qquad \text{III-6}$$

$$\text{Covar } (\hat{D},\hat{k}) = -Dk \; \frac{6}{n(n+1)} \qquad \text{III-7}$$

For evaluation $\hat{D}$ and $\hat{k}$ would be used.